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**Existence Results for a Second Order Differential
Equations with Nonlocal Conditions**

Eduardo Hernández M.

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Existência de Soluções para uma Equação Diferencial de Segunda Ordem com Condição não Local.

Eduardo Hernández M.

Departamento de Matemática - ICMC - USP.

Resumo

Neste artigo mostramos a existência de soluções fracas e clássicas para um problema de segunda ordem com condição não local modelado na forma

$$\begin{aligned}\ddot{u}(t) &= Au(t) + f(t, u(t), \dot{u}(t)), & t \in (-T_0, T_1), \\ u(0) &= x_0 + q(t_1, t_2, \dots, t_n : u(\cdot), \dot{u}(\cdot)), \\ \dot{u}(0) &= y_0 + p(t_1, t_2, \dots, t_n : u(\cdot), \dot{u}(\cdot)),\end{aligned}$$

onde A é o gerador infinitesimal de uma função coseno fortemente continua de operadores lineares num espaço Banach X , $-T_0 < 0 < t_1 < t_2 < \dots < t_n < T_1$; $f : \mathbb{R} \times X \times X \rightarrow X$, $q, p : [-T_0, T_1]^n \times X^2 \rightarrow X$ são apropriadas e o símbolo $h(t_1, t_2, t_3, \dots, t_n, u(\cdot), \dot{u}(\cdot))$ é usado no sentido que “.”, somente pode ser substituído pelos pontos t_i , por exemplo $h(t_1, t_2, t_3, \dots, t_n, u(\cdot), \dot{u}(\cdot)) = \sum_{i=1}^n (\alpha_i u(t_i) + \beta \dot{u}(t_i))$.

Existence Results for a Second Order Differential Equation with Nonlocal Conditions.

Eduardo Hernández M.

Abstract

In this paper we give some existence results for a nonlocal Cauchy problem associated with the second order differential equation $\ddot{u}(t) = Au(t) + f(t, u(t), \dot{u}(t))$, where A is the infinitesimal generator of a strongly continuous cosine family on a Banach space X and f is a appropriate continuous function.

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1 Introduction

This paper is concern with the second order nonlocal Cauchy problem

$$\begin{cases} \ddot{u}(t) = Au(t) + f(t, u(t), \dot{u}(t)), & t \in (-T_0, T_1), \\ u(0) = x_0 + q(t_1, t_2, \dots, t_n : u(\cdot), \dot{u}(\cdot)), \\ \dot{u}(0) = y_0 + p(t_1, t_2, \dots, t_n : u(\cdot), \dot{u}(\cdot)), \end{cases} \quad (1)$$

where A is the infinitesimal generator of a strongly continuous cosine function of linear operators, $(C(t))_{t \in \mathbb{R}}$, on a Banach space X ; $-T_0 < 0 < t_1 < t_2 < \dots < t_n < T_1$; $f : \mathbb{R} \times X \times X \rightarrow X$, $q, p : [-T_0, T_1]^n \times X^2 \rightarrow X$ are appropriated functions and the symbol $h(t_1, t_2, t_3, \dots, t_n, u(\cdot), \dot{u}(\cdot))$ is used in the sense that “.” can be substitute only for the points t_i , for instance $h(t_1, t_2, t_3, \dots, t_n, u(\cdot), \dot{u}(\cdot)) = \sum_{i=1}^n (\alpha_i u(t_i) + \beta_i \dot{u}(t_i))$.

Motivated for numerous applications Byszewski studied in [1], the first order evolution nonlocal problem

$$\begin{cases} \dot{u}(t) = Au(t) + f(t, u(t)), & t \in (-T_0, T_1), \\ u(0) = x_0 + q(t_1, t_2, \dots, t_n : u(\cdot)), \end{cases} \quad (2)$$

where A is the infinitesimal generator of a C_0 -semigroup of linear operators on a Banach space X . In the cited paper, Byszewski prove the existence of mild, strong and classical solutions for (2), employing usual assumptions on f and the semigroup theory. For a more complete reference for differential equations with nonlocal conditions refer to [1]-[4] for the first order case and to [5]-[6] for the second order problem.

Our goal will be give some existence results of mild and classical solutions for the abstract nonlocal Cauchy problem (1) using the theory of cosine functions.

Throughout this paper, X will be a Banach space provided with a norm $\|\cdot\|$; $A : D(A) \rightarrow X$ will be the infinitesimal generator of strongly continuous cosine function of linear operators, $C = (C(t))_{t \in \mathbb{R}}$, on X and $S = (S(t))_{t \in \mathbb{R}}$ will be the associated sine function, that is $S(t)x = \int_0^t C(s)x ds$, $x \in X$. For the theory of cosine functions, refer Fatorinni [7] and Travis [8]. We will point out here some notations and properties that will be essential for us. Along of this paper we denote by $D(A)$ and E the sets;

$$D(A) = \{x \in X : C(t)x \text{ is twice continuously differentiable}\},$$

$$E = \{x \in X : C(t)x \text{ is once continuously differentiable}\}.$$

The properties in the next result are well know ([8, pp.77]).

Lemma 1.1 *In the previous condition, the next properties hold.*

1. If $x \in X$ then $S(t)x \in E$.
2. If $x \in E$ then $S(t)x \in D(A)$, $\frac{d}{dt}C(t)x = AS(t)x$ and $\frac{d^2}{dt^2}S(t)x = AS(t)x$.
3. If $x \in D(A)$ then $C(t)x \in D(A)$ and $\frac{d^2}{dt^2}C(t)x = AC(t)x = C(t)Ax$.
4. If $x \in D(A)$ then $S(t)x \in D(A)$ and $\frac{d^2}{dt^2}S(t)x = AS(t)x = S(t)Ax$.

This work contain two sections. In section 2 we discuss the existence of solution for the abstract nonlocal problem (1). Firstly, we introduce the concept of mild and classical solution and subsequently employing the contraction mapping principle and the ideas in [8], we prove the existence of mild and classical solutions.

The terminology and notations are those generally used in operator theory. In particular, if Z is a Banach space, we indicate by $\mathcal{L}(Z)$ the Banach space of the bounded linear operators from Z into Z . In addition $B_r[x : Z]$ will denote the closed ball in Z with center at x and radius r .

Finally, for a nonnegative bounded function $\xi : (-T_0, T_1) \rightarrow \mathbb{R}$ we will employ the notation ξ_t , $t \in (-T_0, T_1)$, for $\xi_t = \sup\{\xi(\theta) : \theta \in (-T_0, t]\}$.

Remark 1.2 *In the rest of this work, we use the notations $q(t_i : x, y)$ and $p(t_i : x, y)$ for $q(t_1, t_2, \dots, t_n : x, y)$ and $p(t_1, t_2, \dots, t_n : x, y)$ respectively.*

2 Existence Results

In this section we discuss the existence of mild and classical solutions for the abstract nonlocal Cauchy problem (1). By comparison with Travis [8], we introduce the following definitions:

Definition 2.1 *A function $u \in C^1((-T_0, T_1) : X)$ is a mild of the nonlocal Cauchy problem (1), if*

$$u(t) = C(t)(x_0 + q(t_i : u, \dot{u})) + S(t)(y_0 + p(t_i : u, \dot{u})) + \int_0^t S(t-s)f(s, u(s), \dot{u}(s))ds$$

for every $t \in (-T_0, T_1)$.

Definition 2.2 A function $u(\cdot) \in C^2((-T_0, T_1) : X)$ is a classical solution of the nonlocal Cauchy problem (1), if $u(\cdot)$ is solution of (1).

Next, we show the existence and uniqueness of mild solutions using the contraction mapping principle, for this reason we introduce the following technical assumptions.

Assumptions : In the following assumptions r_1, r_2 are fixed positive numbers.

(A₁) The function f is continuous and there exist positive constants $l^1(f)$ and $l^2(f)$ such that

$$\| f(t, x, y) - f(t, x', y') \| \leq l^1(f) \| x - x' \| + l^2(f) \| y - y' \|,$$

for every $x, x' \in B_{r_1}[0, X]$ and $y, y' \in B_{r_2}[0, X]$.

(A₂) The functions $q(t_i : \cdot), p(t_i : \cdot) : [-T_0, T_1]^n \times X^2 \rightarrow X$ are continuous and there exist positive constants $l^i(p), l^i(q)$ such that

$$\begin{aligned} \| q(t_i : u, \tilde{u}) - q(t_i : v, \tilde{v}) \| &\leq l^1(q) \| u - \tilde{u} \|_{T_1} + l^2(q) \| v - \tilde{v} \|_{T_1}, \\ \| p(t_i : u, \tilde{u}) - p(t_i : v, \tilde{v}) \| &\leq l^1(p) \| u - \tilde{u} \|_{T_1} + l^2(p) \| v - \tilde{v} \|_{T_1}, \end{aligned}$$

for every $(u, \tilde{u}), (v, \tilde{v})$ in $C((-T_0, T_1) : B_{r_1}[0, X] \times B_{r_2}[0, X])$

(A₃) There exist functions $B : [-T_0, T_1] \rightarrow \mathcal{L}(X)$ and $g : [-T_0, T_1]^n \times X^2 \rightarrow X$ such that

- i) $B(0) = 0$, $B(\cdot)$ is strongly continuous and $\frac{d}{dt}C(t)q(t_i : x, y) = B(t)g(t_i : x, y)$ for every $x, y \in X$,
- ii) g is continuous and there exist positive constants $l^i(g)$ such that

$$\| g(t_i : u, v) - g(t_i : \tilde{u}, \tilde{v}) \| \leq l^1(g) \| u - \tilde{u} \|_{T_1} + l^2(g) \| v - \tilde{v} \|_{T_1},$$

for every $(u, \tilde{u}), (v, \tilde{v}) \in C((-T_0, T_1) : B_{r_1}[0, X] \times B_{r_2}[0, X])$.

Remark 2.3 In the proof of the following theorem we will employ the notations N_m , $m = p, q, g$, for

$$N_m = \sup\{m(t_i : u, v) : (u, v) \in C((-T_0, T_1) : B_{r_1}[0, X] \times B_{r_2}[0, X])\}$$

and N_f for the sup of the function f on $[-T_0, T_1] \times B_{r_1}[0, X] \times B_{r_2}[0, X]$.

Theorem 2.4 Let $x_0 \in E \cap B_{r_1}(0, X)$, $y_0 \in B_{r_2}(0, X)$ and assume that assumption A₁-A₃ holds. If

$$\| C \|_{T_1} (\| x_0 \| + N_q) + \| S \|_{T_1} (\| y_0 \| + N_p + N_f(T_0 + T_1)) \leq r_1, \quad (3)$$

$$\| AS(\cdot)x_0 \|_{T_1} + \| B \|_{T_1} N_g + \| C \|_{T_1} (\| y_0 \| + N_p + N_f(T_0 + T_1)) \leq r_2. \quad (4)$$

and

$$\| C \|_{T_1} l^i(q) + (\| C \|_{T_1} + \| S \|_{T_1})(l^i(p) + (T_0 + T_1)l^i(f)) + \| B \|_{T_1} l^i(g) < 1, \quad i = 1, 2$$

then there exists a unique mild solution of (1).

Proof: Let $Z = \{(u, v) : u, v \in C((-T_0, T_1) : X), \dot{u} = v\}$ endowed with the norm $\|(u, v)\|_1 = \|u\|_{T_1} + \|v\|_{T_1}$ and $Y = \{(u, v) \in Z : \|u\|_{T_1} \leq r_1, \|v\|_{T_1} \leq r_2\}$. Clearly Z is a Banach space and Y a closed subset of Z . On the space Y we define the mapping $\Phi : (u, v) \rightarrow (\Phi_1(u), \Phi_2(u))$ by

$$\begin{aligned}\Phi_1(u)(t) &= C(t)(x_0 + q(t_i : u, \dot{u})) + S(t)(y_0 + p(t_i : u, \dot{u})) \\ &\quad + \int_0^t S(t-s)f(s, u(s), \dot{u}(s))ds, \\ \Phi_2(u)(t) &= AS(t)x_0 + B(t)g(t_i : u, \dot{u}) + C(t)(y_0 + p(t_i : u, \dot{u})) \\ &\quad + \int_0^t C(t-s)f(s, u(s), \dot{u}(s))ds.\end{aligned}$$

Next we'll prove that Φ is a contraction mapping on Y . A simple estimate show that for $(u, v) \in Y$

$$\|\Phi_1(u)(t)\| \leq \|C\|_{T_1} (\|x_0\| + N_q) + \|S\|_{T_1} (\|y_0\| + N_p + N_f(T_0 + T_1)), \quad (5)$$

$$\|\Phi_2(u)(t)\| \leq \|ASx_0\|_{T_1} + \|B\|_{T_1} N_g + \|C\|_{T_1} (\|y_0\| + N_p + N_f(T_0 + T_1)), \quad (6)$$

thus, from (3)-(4) and (5)-(6) follow that $\Phi(u, v) \in Y$.

To prove that Φ is a contraction, we take $(u, w), (v, z)$ in Y . Then for $t \in (-T_0, T_1)$ we see that can be compute

$$\begin{aligned}\|\Phi_1(u)(t) - \Phi_1(v)(t)\| &\leq \|C\|_{T_1} (l^1(q) \|u - v\|_{T_1} + l^2(q) \|\dot{u} - \dot{v}\|_{T_1}) \\ &\quad + \|S\|_{T_1} (l^1(p) \|u - v\|_{T_1} + l^2(p) \|\dot{u} - \dot{v}\|_{T_1}) \\ &\quad + \int_0^t \|S\|_{T_1} (l^1(f) \|u - v\|_{\theta} + l^2(f) \|\dot{u} - \dot{v}\|_{\theta}) d\theta,\end{aligned}$$

thus

$$\begin{aligned}\|\Phi_1(u) - \Phi_1(v)\|_{T_1} &\leq (\|C\|_{T_1} l^1(q) + \|S\|_{T_1} (l^1(p) + (T_0 + T_1)l^1(f))) \|u - v\|_{T_1} \\ &\quad + (\|C\|_{T_1} l^2(q) + \|S\|_{T_1} (l^2(p) + (T_0 + T_1)l^2(f))) \|\dot{u} - \dot{v}\|_{T_1}.\end{aligned} \quad (7)$$

Similarly

$$\begin{aligned}\|\Phi_2(u) - \Phi_2(v)\|_{T_1} &\leq (\|B\|_{T_1} l^1(g) + \|C\|_{T_1} (l^1(p) + (T_0 + T_1)l^1(f))) \|u - v\|_{T_1} \\ &\quad + (\|B\|_{T_1} l^2(g) + \|C\|_{T_1} (l^2(p) + (T_0 + T_1)l^2(f))) \|\dot{u} - \dot{v}\|_{T_1}.\end{aligned} \quad (8)$$

From (7)-(8)

$$\|\Phi(u, w) - \Phi(v, z)\|_1 \leq \Theta \|(u, w) - (v, z)\|_1,$$

and thus, Φ is a contraction. Clearly the unique fix point of Φ is the unique mild solution of (1). The proof is complete.

Now we consider, briefly, some simple examples of functions satisfying $(\mathbf{A}_3)$. In the next examples, we said that $h(t_i : \cdot) : [0, T]^n \times Z \rightarrow W$ satisfies the condition $\mathbf{H}(\mathbf{Z}, \mathbf{W})$ if exist positive constants $l^i(h)$ such that

$$\| h(t_i : u, \tilde{u}) - h(t_i : v, \tilde{v}) \|_W \leq l^1(h) \| u - v \|_Z + l^2(h) \| \tilde{u} - \tilde{v} \|_Z,$$

for every $(u, \tilde{u}), (v, \tilde{v})$ in $C((-T_0, T_1) : Z \times Z)$.

- 1) Let $[D(A)]$ be the space $D(A)$ endowed with the graph norm. If $h(t_i : \cdot)$ verifies the condition $\mathbf{H}(\mathbf{X}, [\mathbf{D}(\mathbf{A})])$ then $h(t_i : \cdot)$ verifies $(\mathbf{A}_3)$ since,

$$\frac{d}{dt} C(t) h(t_i : x, y) = AS(t) h(t_i : x, y) = S(t) A h(t_i : x, y),$$

for each $x, y \in X$.

- 2) Assume that $(C(t))_{t \in \mathbb{R}}$ satisfies condition F (see [8, pp.81] for details). If $[D(B)]$ is the space $D(B)$ endowed with the graph norm and $h(t_i : \cdot)$ verifies the condition $\mathbf{H}(\mathbf{X}, [\mathbf{D}(\mathbf{B})])$ then $h(t_i : \cdot)$ verifies $(\mathbf{A}_3)$. The assertion is clear since $BS(\cdot)$ is uniformly bounded on $[-T_0, T_1]$ and

$$\frac{d}{dt} C(t) h(t_i : x, y) = AS(t) h(t_i : x, y) = BS(t) B h(t_i : x, y)$$

for every $x, y \in X$.

- 3) Let $r \in \mathbb{R}$ and $h(t_i : \cdot)$ verifying condition $\mathbf{H}(\mathbf{X}, \mathbf{X})$. From Proposition 2.2 in [8] we have that,

$$\frac{d}{dt} C(t) S(r) h(t_i : x, y) = AS(t) S(r) h(t_i : x, y) = \frac{1}{2} (C(r+t) - C(r-t)) h(t_i : x, y)$$

for each $x, y \in X$, thus $q(t_i : x, y) = S(r) h(t_i : x, y)$ verifies $\mathbf{A}_3$.

2.1 Classical Solutions

The proof of the following theorem is similar to the proof of Proposition 3.2 in Travis [8] and for this reason we only consider the principal steps of the proof by completeness.

Theorem 2.5 (*Classical solution*) *Let assumptions in Theorem 2.4 be satisfied. Assume that f is continuously differentiable and that exists $l^3(f) > 0$ such that*

$$\max\{ \| f_{x_j}(t, x, y) - f_{x_j}(t, x, \tilde{y}) \| : j = 1, 2, 3 \} \leq l^3(f) \| y - \tilde{y} \|, \quad (9)$$

for every $(x, y) \in B_{r_1}[0, X] \times X$.

If $u(\cdot)$ is the mild solution of (1) and $(u(0), \dot{u}(0)) \in D(A) \times E$ and $\dot{u}(0) \in E$ then $u(\cdot)$ is a classical solution.

Proof: Since $f_{x_3}(s, u(s), \dot{u}(s))$ is bounded on $(-T_0, T_1)$, from the contraction mapping principle there exists a unique solution, $v \in C((-T_0, T_1) : X)$, of the integral equation

$$\begin{aligned} v(t) &= C(t)(Au(0) + f(0, u(0), \dot{u}(0))) + AS(t)(\dot{u}(0)) \\ &+ \int_0^t C(t-s) (f_{x_1}(s, u(s), \dot{u}(s)) + f_{x_2}(s, u(s), \dot{u}(s))\dot{u}(s) + f_{x_3}(s, u(s), \dot{u}(s))v(s)) ds \end{aligned}$$

From the steps in the proof of Proposition 3.3 in [8], follow that $z(t) = \dot{u}(0) + \int_0^t v(s)ds = \dot{u}(t)$, thus $s \rightarrow f(s, u(s), \dot{u}(s))$ is once continuously differentiable on $(-T_0, T_1)$. From Proposition 2.4 in Travis [8], there exist a unique classical solution, $w : (-T_0, T_1) \rightarrow X$, of

$$\begin{aligned} \ddot{w}(t) &= Aw(t) + f(t, u(t), \dot{u}(t)), & t \in (-T_0, T_1), \\ w(0) &= x_0 + q(t_i : u, \dot{u}) = u(0), \\ \dot{w}(0) &= y_0 + p(t_i : u, \dot{u}) = \dot{u}(0), \end{aligned} \tag{10}$$

moreover

$$w(t) = C(t)(x_0 + q(t_i : u, \dot{u})) + S(t)(y_0 + p(t_i : u, \dot{u})) + \int_0^t S(t-s)f(s, u(s), \dot{u}(s))ds.$$

Obviously $u = w$ and therefore u is a classical solution. The proof is complete.

The methods used in the proofs of the previous theorems can be used to show similar existence results for

$$\begin{cases} \ddot{u}(t) &= Au(t) + f(t, u(t)), & t \in (-T_0, T_1), \\ u(0) &= x_0 + q(t_i : u), \\ \dot{u}(0) &= y_0 + p(t_i : u). \end{cases} \tag{11}$$

Definition 2.6 A function $u \in C^1((-T_0, T_1) : X)$ is a mild solution of (11) if

$$u(t) = C(t)(x_0 + q(t_i : u)) + S(t)(y_0 + p(t_i : u)) + \int_0^t S(t-s)f(s, u(s))ds,$$

for every $t \in (-T_0, T_1)$.

Definition 2.7 A function $u \in C^2((-T_0, T_1) : X)$ is a classical solution of the nonlocal Cauchy problem (11), if u is solution of (11).

Assumption : In the following assumption, r_1 is a fixed positive constant.

(A₄) There exist positive constants $l(f), l(q), l(p)$ such that

$$\begin{aligned} \|f(t, x) - f(t, y)\| &\leq l(f) \|x - y\|, \\ \|q(t_i : u) - q(t_i : v)\| &\leq l(q) \|u - v\|_{T_1}, \\ \|p(t_i : u) - p(t_i : v)\| &\leq l(p) \|u - v\|_{T_1}, \end{aligned}$$

for every $(x, y) \in B_{r_1}[0, X] \times B_{r_1}[0, X]$, $(u, v) \in C((-T_0, T_1) : B_{r_1}[0, X] \times B_{r_1}[0, X])$.

Theorem 2.8 Assume that assumption (A₄) hold and that

$$\begin{aligned} \|C\|_{T_1} (\|x_0\| + N_q) + \|S\|_{T_1} (\|y_0\| + N_p + N_f(T_0 + T_1)) &\leq r_1, \\ \|C\|_{T_1} l(q) + \|S\|_{T_1} (l(f)(T_0 + T_1) + l(p)) &< 1. \end{aligned}$$

Then there exist a unique mild solution of the nonlocal Cauchy problem (11).

Theorem 2.9 Assume that assumption in Theorem 2.8 hold and that f is continuously differentiable. If $u(\cdot)$ is the mild solution of (11) and $(u(0), \dot{u}(0)) \in D(A) \times E$ then $u(\cdot)$ is a classical solution.

References

- [1] L. Byszewski, Theorems about the existence and uniqueness of solutions of a semilinear evolution nonlocal Cauchy problem. *J. Math. Anal. Appl.* **162** 2 494-505 1991.
- [2] L. Byszewski; H. Akca, Existence of solutions of a semilinear functional-differential evolution nonlocal problem. *Nonlinear Anal.* **34** 1 65-72 1998.
- [3] L. Byszewski; Lakshmikantham, V, Theorem about the existence and uniqueness of a solution of a nonlocal abstract Cauchy problem in a Banach space. *Appl. Anal.* **40** 1 11-19 1991.
- [4] L. Byszewski; H. Akca, On a mild solution of a semilinear functional-differential evolution nonlocal problem. *J. Appl. Math. Stochastic Anal.* **10** 3 265-271 1997.
- [5] S. K. Ntouyas; P. Ch. Tsamatos, Global existence for second order semilinear ordinary and delay integrodifferential equations with nonlocal conditions. *Appl. Anal.* **67** 3-4 245-257 (1997).
- [6] S. K. Ntouyas, Global existence results for certain second order delay integrodifferential equations with nonlocal conditions. *Dynam. Systems Appl.* **7** 3 415-425 1998.
- [7] H.O. Fattorini. *Second Order Linear Differential Equations in Banach Spaces*. North-Holland. Mathematics Studies 108.
- [8] C. Travis; G. Webb, Cosine families and abstract nonlinear second order differential equations. *Acta Math. Acad. Sci. Hungar.* **32** 1-2, 75-96 1978.

Eduardo Hernández M.
Departamento de Matemática
Instituto de Ciências Matemáticas e de Computação
Universidade de São Paulo - Campus de São Carlos
Caixa Postal 668
13560-970 São Carlos, SP. Brazil
lalohm@icmc.sc.usp.br

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