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A Remark on Neutral Partial Differential Equations

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Uma Nota Sobre Equações Funcionais do Tipo Neutro

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Resumo

Neste artigo fazemos algumas observações respeito de varios recentes trabalhos estudando diferentes problemas diferenciais relacionados com a equação de tipo neutro

$$\begin{aligned} \frac{d}{dt}(x(t) + G(t, x_t)) &= Ax(t) + F(t, x_t), \quad t \in I = [0, a], \\ x_0 &= \varphi \in \mathcal{D}, \end{aligned}$$

sendo A o gerador infinitesimal de um semigrupo fortemente contínuo de operadores lineares definidos em um espaço de Banach abstrato X; x_t a historia da solução no instante t; $\mathcal{D}$ o espaço de fase e $F(\cdot), G(\cdot)$ funções apropriadas.

A Remark on Neutral Partial Differential Equations

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Abstract

In this work we make some observations with respect to some recent papers reporting existence of solutions of partial neutral functional differential equations.

1 Introduction

The papers [1], [2], [3], [4], [5], [6], [7], [8], [15] reported existence results of mild solutions of some different " partial " functional differential problems related to a class of partial neutral functional differential equation described in the form

$$\begin{aligned} \frac{d}{dt}(x(t)+G(t,x_t)) &= Ax(t)+F(t,x_t), \quad t\in I=[0,a], \\ x_0 &= \varphi\in\mathcal{D}, \end{aligned}$$

where A is the infinitesimal generator of a compact semigroup of bounded linear operators, $(T(t))_{t\geq 0}$, on a Banach space X and the histories, x_t, belong to a appropriate phase space $\mathcal{D}$. However, in these works their authors impose the next restrictive compactness assumptions on the operator family generated by A.

Assumption: " *A is the infinitesimal generator of a compact C_0-semigroup of bounded linear operators $(T(t))_{t\geq 0}$ on a Banach space X and there exist positive constants M_1, M_2 such that*

$$\| T(t) \|\leq M_1 \quad \textit{and} \quad \| AT(t) \|\leq M_2, \quad t\in(0,a]."$$

It's easy to prove that the previous assumption imply that the underlying space X has finite dimension. As consequence, the equations treated in these works are really ordinary and not partial equations.

Our objetive in this work is alert respect of these papers and by mean of the study of the existence of mild solutions of the abstract Cauchy problem

$$\frac{d}{dt}(x(t)+g(t,x(t))) = Ax(t)+f(t,x(t)), \quad t\in I=[0,a], \tag{1}$$

$$x(0) = x_0\in X, \tag{2}$$

give some simples ideas for the reformulation of these results.

By considering the technical approach of the refereed papers, we always assume that in the system (1)-(2), $A : D(A) \to X$ is the infinitesimal generator of an uniformly bounded analytic semigroup of linear operators, $(T(t))_{t\geq 0}$, on a Banach space X and that $f, g : I \times X \to X$ are appropriate functions.

Next we mention a few results and notations needed to establish our results. We will assume that $0 \in \rho(A)$ and that $\| T(t) \| \leq \tilde{M}$ for all $t \in I$. Under these conditions, it's possible to define the fractional power $(-A)^\alpha$, $0 < \alpha \leq 1$, as a closed linear operator on its domain $D(-A)^\alpha$. Furthermore, $D(-A)^\alpha$ is dense in X and the expression $\| x \|_\alpha = \| (-A)^\alpha x \|$ defines a norm in $D(-A)^\alpha$. The following properties are well known, see [13], pp. 74.

Lemma 1 *Let $0 < \beta \leq \alpha \leq 1$. Under the previous conditions, the following properties hold:*

(i) *X_α is a Banach space.*

(ii) *The inclusion $X_\alpha \to X_\beta$ is continuous.*

(iii) *There exists $C_\alpha > 0$ such that $\| (-A)^\alpha T(t) \| \leq \frac{C_\alpha}{t^\alpha}$, for $t > 0$.*

For a complementary literature respect of semigroup theory, we refer to the reader to Pazy [13].

The terminologies and notations are those generally used in functional analysis. In particular, $C(I; X)$ is the space of the continuous function from I into X endowed with the topology of the uniform convergence and $B_r(x : Z)$ denotes the closed ball with center at x and radius $r > 0$ in an metric space Z. Additionally, for a bounded function $\xi : [0, a] \to [0, \infty)$ and $0 \leq t \leq a$ we will employ the notation ξ_t for

$$\xi_t = \sup\{\xi(s) : s \in [0, t]\}. \tag{3}$$

The paper has two sections. In section 2 we establish some existence results of mild solution of the system (1)-(2) using the Theorems 1 and 2.

To finish this introduction, we observe that our results will proved using the following well known results.

Theorem 1 *[9, pp. 61] Let D be a convex subset of a Banach space X and assume that $0 \in D$. Let $F : D \to D$ be a completely continuous map. Then the map F has a fix point in D or the set $\{x \in D : x = \lambda F(x),\ 0 < \lambda < 1\}$ is unbounded.*

Theorem 2 *(Sadovskii [14]) Let D be a convex, closed and bounded subset of a Banach space X. If $F : D \to D$ is a condensing operator, then F has a fix point in D.*

2 Existence Results

In this section we discuss the existence of mild solutions of the system (1)-(2). At first we introduce the following definition.

Definition 1 *A function $x : [0,a] \to X$ is a mild solution of (1)-(2), if $x(0) = x_0$; the function $s \to AT(t-s)g(s,x(s))$ is integrable on $[0,t)$ for each $0 \le t < a$ and*

$$
\begin{aligned}
x(t) &= T(t)(x_0 + g(0,x_0)) - g(t,x(t)) - \int_0^t AT(t-s)g(s,x(s))ds \\
&\quad + \int_0^t T(t-s)f(s,x(s))ds, \qquad t \in I.
\end{aligned}
$$

To study the existence of mild solutions of (1)-(2), we consider the following conditions.

$\mathbf{H_1}$ The function $f : I \times X \to X$ satisfies the following conditions:

(i) The function $f(t,\cdot) : X \to X$ is continuous $a.e.$ $t \in I$.

(ii) For each $x \in X$, the function $f(\cdot,x) : I \to X$ is strongly measurable.

(iii) There exist a continuous function $m : I \to [0,\infty)$ and a continuous nondecreasing function $W : [0,\infty) \to (0,\infty)$ such that

$$\| f(t,x) \| \le m(t)W(\| x \|), \qquad (t,x) \in I \times X.$$

$\mathbf{H_2}$ There exist constants $0 < \beta < 1$, c_1, c_2 such that g is continuous with values in X_β and

$$\| (-A)^\beta g(t,x) \| \le c_1 \| x \| + c_2, \qquad (t,x) \in I \times X.$$

$\mathbf{H_3}$ The set of functions $\{t \to g(t,x(t)) : x \in B_r(0, C(I : X))\}$ is equicontinuous on I for every $r > 0$.

Theorem 3 *Let assumptions $\mathbf{H_1}$-$\mathbf{H_3}$ be satisfied and assume that*

(a) *For every $\epsilon > 0$ and all $r > 0$ there are compact sets $W^i_{\epsilon,r} \subset X$, $i = 1,2$, such that $T(\epsilon)(-A)^\beta g(s,x) \in W^1_{\epsilon,r}$ and $T(\epsilon)f(s,x) \in W^2_{\epsilon,r}$ for every $(s,x) \in I \times B_r(0,X)$.*

If $\mu = c_1(\| (-A)^{-\beta} \| + \frac{C_{1-\beta}a^\beta}{\beta}) < 1$ and $\frac{\tilde{M}}{1-\mu}\int_0^a m(s)ds < \int_c^\infty \frac{1}{W(s)}ds$ where

$$c = \frac{1}{1-\mu}\left[\tilde{M}(\| x_0 \| (c_1 \| (-A)^{-\beta} \| + 1) + c_2 \| (-A)^{-\beta} \|) + c_2(\| (-A)^{-\beta} \| + C_{1-\beta}\frac{a^\beta}{\beta})\right],$$

then there exists a mild solution of the abstract Cauchy problem (1)-(2).

Proof: Let $\Gamma : C(I : X) \to C(I : X)$ be the map defined by

$$
\begin{aligned}
\Gamma x(t) &= T(t)(x_0 + g(0,x_0)) - g(t,x(t)) - \int_0^t AT(t-s)g(s,x(s))ds \\
&\quad + \int_0^t T(t-s)f(s,x(s))ds, \quad t \in I.
\end{aligned}
$$

From the assumptions, we know that that both, $s \to (-A)^\beta g(s,x(s))$ and $s \to g(s,x(s))$ are continuous. In addition, in view of the fact that $T(\cdot)$ is an analytic semigroups (see

[13]), the operator function $s \to AT(t-s)$ is continuous in the uniform operator topology on $[0,t)$ which from the estimate

$$\| (-A)T(t-s)g(s,x(s)) \| = \| (-A)^{1-\beta}T(t-s)(-A)^{\beta}g(s,x(s)) \| \leq \frac{C_{1-\beta}Cte}{(t-s)^{1-\beta}},$$

and the Bochner's Theorem imply that $\| AT(t-s)g(s,x(s)) \|$ is integrable on $[0,t)$. Now, it's clear that Γ is well defined and with values in $C(I:X)$. Moreover, from the Lebesgue dominated convergence theorem we infer that Γ is continuous.

In order to use the Theorem 1, we obtain *a priori* estimates for the solutions of the integral equation $x = \lambda\Gamma x$, $\lambda \in (0,1)$. Let $\lambda \in (0,1)$ and $x^{\lambda}(\cdot)$ be a solution of $x = \lambda\Gamma x$. From the definition of Γ we get

$$\begin{aligned}
\| x^{\lambda}(t) \| \leq\ & \tilde{M}(\| x_0 \| (\| (-A)^{-\beta} \| c_1 + 1) + \| (-A)^{-\beta} \| c_2) + \| (-A)^{-\beta} \| c_1 \| x^{\lambda}(t) \| \\
& + \| (-A)^{-\beta} \| c_2 + C_{1-\beta}c_1 \int_0^t \frac{\| x^{\lambda} \|_s}{(t-s)^{1-\beta}} ds + \frac{C_{1-\beta}c_2 a^{\beta}}{\beta} \\
& + \tilde{M} \int_0^t m(s)W(\| x^{\lambda} \|_s) ds
\end{aligned}$$

where the notation introduced in (3) has been used. Consequently,

$$\begin{aligned}
\| x^{\lambda} \|_t \leq\ & \tilde{M}(\| x_0 \| (\| (-A)^{-\beta} \| c_1 + 1) + \| (-A)^{-\beta} \| c_2) + c_2(\| (-A)^{-\beta} \| + \frac{C_{1-\beta}a^{\beta}}{\beta}) \\
& + \mu \| x^{\lambda} \|_t + \tilde{M} \int_0^t m(s)W(\| x^{\lambda} \|_s) ds,
\end{aligned}$$

and then

$$\| x^{\lambda} \|_t \leq c + \frac{\tilde{M}}{1-\mu} \int_0^t m(s)W(\| x^{\lambda} \|_s) ds, \qquad t \in I.$$

Designating by $\beta^{\lambda}(t)$ the right hand side of the above expression we get

$$\beta_{\lambda}'(t) \leq \frac{\tilde{M}}{1-\mu} m(t)W(\beta_{\lambda}(t))$$

and so that

$$\int_c^{\beta_{\lambda}(t)} \frac{ds}{W(s)} \leq \frac{\tilde{M}}{1-\mu} \int_0^t m(s) ds < \int_c^{\infty} \frac{ds}{W(s)},$$

which imply that the functions $\beta^{\lambda}(\cdot)$ are uniformly bounded on I. Thus, the functions $x^{\lambda}(\cdot)$ are uniformly bounded on I.

Next we prove that Γ is completely continuous. To this end, by using the relation $A\int_0^t T(s)x ds = T(t)x - x$, we rewrite Γ in the form $\Gamma = \sum_{i=1}^{3} \Gamma_i$ where

$$\begin{aligned}
\Gamma_1 x(t) &= T(t)(x_0 + g(0,x_0) - g(t,x(t))), \\
\Gamma_2 x(t) &= -\int_0^t AT(t-s)g(s,x(s)) ds, \\
\Gamma_3 x(t) &= \int_0^t T(t-s)f(s,x(s)) ds + \int_0^t AT(t-s)g(t,x(t)) ds.
\end{aligned}$$

From the Lebesgue dominated convergence theorem and the conditions $\mathbf{H_1}, \mathbf{H_2}$, follows that each Γ_i is continuous. In the sequel we prove that $\Gamma_i(B_r(0, C(I:X)))$ is relatively compact in $C(I:X)$ for every $i = 1, .., 3$. In what follow $B_r = B_r(0, C(I:X))$.

Step 1. The set $\Gamma_1(B_r)$ is relatively compact in $C(I:X)$.

Firstly we study the equicontinuity of $\Gamma_1(B_r)$. Let $0 < \epsilon < t_0 < t \leq a$ and $W^1_{\epsilon,r}$ be the compact set in (a). Using the condition $\mathbf{H_3}$ and the fact that set of functions $\{s \to T(s)x : x \in W^1_{\epsilon,r}\}$ is equicontinuous on $[0, a]$, we can choose $0 < \delta < \epsilon$ such that

$$\begin{aligned} \| T(t)x - T(t_0)x \| &< \epsilon, \qquad x \in W^1_{\epsilon,r}, \\ \| g(t, x(t)) - g(t_0, x(t_0)) \| &< \epsilon, \qquad x(\cdot) \in B_r, \end{aligned}$$

when $| t - t_0 | < \delta$. Under these conditions, for $x(\cdot) \in B_r$ and $| t - t_0 | < \delta$ we get

$$\begin{aligned} \| \Gamma_1 x(t) - \Gamma_1 x(t_0) \| &\leq \| (T(t_0) - T(t))(x_0 + g(0, x_0)) \| \\ &\quad + \| T(t_0) \| \| g(t_0, x(t_0)) - g(t, x(t)) \| \\ &\quad + \| (-A)^{-\beta}(T(t_0 - \epsilon) - T(t - \epsilon))T(\epsilon)(-A)^{\beta} g(t, x(t)) \| \\ &\leq \| (T(t_0) - T(t))(x_0 + g(0, x_0)) \| + (\| T(t_0) \| + \| (-A)^{-\beta} \|)\epsilon, \end{aligned}$$

which imply that $\Gamma_1(B_r)$ is equicontinuous from the right hand side at t_0. Similarly we can prove the equicontinuity from the left hand side at t_0 and the right equicontinuity at zero. Thus, the set $\Gamma_1(B_r)$ is equicontinuous on I.

It's clear that $\Gamma_1(B_r)(0)$ is compact in X. Moreover, if $0 < \epsilon < t$, we get

$$\Gamma_1(B_r)(t) = \{\Gamma_1 u(t) : u \in B_r\} \subset T(t)(x_0 + g(0, x_0)) + (-A)^{-\beta} T(t - \epsilon) W^1_{\epsilon,r},$$

which proves that $\Gamma_1(B_r)(t)$ is relatively compact in X since $(-A)^{-\beta}T(t-\epsilon)$ is continuous. Thus, the set $\Gamma_1(B_r)$ is relatively compact in $C(I:X)$.

Step 2. The set $\Gamma_2(B_r)$ is relatively compact in $C(I:X)$

At first we prove that $\Gamma_2(B_r)(t) = \{\Gamma_2 u(t) : u \in B_r\}$ is relatively compact in X for every $t \in I$. The case $t = 0$ is trivial. Let $0 < \epsilon < t \leq a$ and $W^1_{\frac{\epsilon}{2},r}$ be the compact set in (a). Since the function $t \to (-A)^{\beta}T(t)$ is continuous in the uniform convergence topology on $[\frac{\epsilon}{2}, a]$, follows that the set $U_\epsilon = \{(-A)^{1-\beta}T(s)x : s \in [\frac{\epsilon}{2}, a],\ x \in W^1_{\frac{\epsilon}{2},r}\}$ is precompact in X. If $x(\cdot) \in B_r$, from the mean value theorem for the Bochner integral, see [12] pp. 25, we get

$$\begin{aligned} \Gamma_2 x(t) &= \int_0^{t-\epsilon} (-A)^{1-\beta} T(t - s - \frac{\epsilon}{2}) T(\frac{\epsilon}{2})(-A)^{\beta} g(s, x(s)) ds \\ &\quad + \int_{t-\epsilon}^{t} (-A)^{1-\beta} T(t - s)(-A)^{\beta} g(s, x(s)) ds \\ &\in (t - \epsilon)\overline{co(U_\epsilon)} + C_\epsilon, \end{aligned}$$

where $co(U_\epsilon)$ denote the convex hull of U_ϵ and $diam(C_\epsilon) \leq 2C_{1-\beta}(c_1 r + c_2)\frac{\epsilon^{\beta}}{\beta}$. These remark permit infer that $\Gamma_2(B_r)(t)$ is relatively compact in X.

Now we prove that $\Gamma_2(B_r)$ is equicontinuous on I. Let $0 \leq t_0 \leq t \leq a$ and $x \in B_r$. Then

$$\begin{aligned}
\| \Gamma_2 x(t) - \Gamma_2 x(t_0) \| &= \| \int_0^t AT(t-s)g(s,x(s))ds - \int_0^{t_0} AT(t_0-s)g(s,x(s))ds \| \\
&= \| (T(t-t_0)-I)\Gamma_2 x(t_0) \| + \int_{t_0}^t \| AT(t-s)g(s,x(s)) \| ds \\
&\leq \| (T(t-t_0)-I)\Gamma_2 x(t_0) \| + C_{1-\beta}(c_1 r + c_2)\frac{(t-t_0)^\beta}{\beta}.
\end{aligned}$$

Since $\Gamma_2(B)(t_0)$ is relatively compact, it is clear that the first term on the right hand side converges to zero when $t \to t_0$ uniformly on $x \in B_r$. This shows that $\Gamma_2(B_r)$ is equicontinuous from the right hand side at t_0. Similarly we can to prove that $\Gamma_2(B_r)$ is equicontinuous from the left hand side at t_0 and equicontinuous at $t = 0$. Thus, $\Gamma_2(B_r)$ is equicontinuous on $[0, a]$ and relatively compact in $C(I : X)$.

Arguing as in step 2, follows that $\Gamma_3(B_r)$ is relatively compact $C(I : X)$. We omit this part of the proof.

These remarks and the Theorem 1 asserts that Γ has a fixed point in $C(I : X)$. This point is a mild solution of the problem (1). The proof is complete. ■

Theorem 4 *Assume that* $\mathbf{H_1}$ *and* $\mathbf{H_2}$ *are verified and that the followings conditions hold.*

(a) *For each* $\epsilon > 0$ *and all* $r > 0$ *there is a compact* $W_{\epsilon,r} \subset X$ *such that* $T(\epsilon)f(s,x) \in W_{\epsilon,r}$ *for every* $(s,x) \in I \times B_r(0,X)$.

(b) *There exist constant* $0 < \beta < 1$ *and* $L_g > 0$ *such that such that* g *is* X_β*-valued and*

$$\| (-A)^\beta g(t,x_1) - (-A)^\beta g(t,x_2) \| \leq L_g \| x_1 - x_2 \|, \qquad (t,x_i) \in I \times X.$$

If $L_g(\| (-A)^{-\beta} \| + \frac{C_{1-\beta}a^\beta}{\beta}) + \liminf_{r \to +\infty} \frac{W(r)}{r} \int_0^a m(s)ds < 1$, *then there exists a mild solution of (1)-(2).*

¶ Let $\Gamma : C(I : X) \to C(I : X)$ be the map defined in the proof of Theorem 3 and consider the decomposition $\Gamma = \tilde{\Gamma}_1 + \tilde{\Gamma}_2$ where

$$\begin{aligned}
\tilde{\Gamma}_1 x(t) &= T(t)(x_0 + g(0,x_0)) - g(t,x(t)) - \int_0^t AT(t-s)g(s,x(s))ds, \quad t \in I, \\
\tilde{\Gamma}_2 x(t) &= \int_0^t T(t-s)f(s,x(s))ds, \quad t \in I.
\end{aligned}$$

We affirm that there exists $r > 0$ such that $\Gamma(B_r(0, C(I : X))) \subset B_r(0, C(I : X))$. In fact, if we assume that the assertion is false, then for every $r > 0$ there exist $x^r \in B_r(0, C(I : X))$ and $t^r \in I$ such that $r < \| \Gamma x^r(t^r) \|$. This yields that

$$\begin{aligned}
r < \| \Gamma x^r(t^r) \| &\leq \tilde{M} \| x_0 + g(0,x_0) \| + \| (-A)^{-\beta} \| L_g \| x(t^r) \| + \| g(t^r, 0) \| \\
&\quad + \int_0^{t^r} \frac{C_{1-\beta}}{(t^r - s)^{1-\beta}} (L_g \| x^r(s) \| + \| g(s,0) \|) ds \\
&\quad + \int_0^{t^r} m(s) W(\| x^r(s) \|) ds
\end{aligned}$$

$$\leq \tilde{M} \parallel x_0 + g(0, x_0) \parallel + \parallel (-A)^{-\beta} \parallel L_g r + \parallel g(t^r, 0) \parallel + \frac{L_g C_{1-\beta} a^\beta r}{\beta}$$
$$+ \int_0^{t^r} \frac{C_{1-\beta} \parallel g(s,0) \parallel}{(t^r - s)^{1-\beta}} ds + W(r) \int_0^a m(s) ds,$$

and so that

$$1 \leq L_g(\parallel (-A)^{-\beta} \parallel + \frac{C_{1-\beta} a^\beta}{\beta}) + \liminf_{r \to +\infty} \frac{W(r)}{r} \int_0^a m(s) ds,$$

which is contradictory with our assumptions.

Let $r_0 > 0$ such that $\Gamma(B_{r_0}(0, C(I : X)) \subset B_{r_0}(0, C(I : X))$. Follows from the proof of Theorem 3 that $\tilde{\Gamma}_2$ is completely continuous on $B_{r_0}(0, C(I : X))$ and from the estimate

$$\parallel \tilde{\Gamma}_1 x - \tilde{\Gamma}_1 y \parallel_a \leq L_g(\parallel (-A)^{-\beta} \parallel + \frac{C_{1-\beta} a^\beta}{\beta}) \parallel x - y \parallel_a,$$

that $\tilde{\Gamma}_1$ is a contraction on $B_{r_0}(0, C(I : X))$. Thus, Γ is a condensing operator on $B_{r_0}(0, C(I : X))$. Now, the assertion is consequence of the Sadovskii point fixed Theorem. ■

References

[1] Balachandran; K. Dauer, J. P. Existence of solutions of nonlinear neutral integrodifferential equations in Banach spaces. *J. Math. Anal. Appl.* 251 (2000), no. 1, 93–105.

[2] Balachandran, K.; Sakthivel, R.; Dauer, J. P. Controllability of neutral functional integrodifferential systems in Banach spaces. *Comput. Math. Appl.* 39 (2000), no. 1-2,

[3] Balachandran, K.; Sakthivel, R. Existence of solutions of neutral functional integrodifferential equation in Banach spaces. *Proc. Indian Acad. Sci. Math. Sci.* 109 (1999), no. 3, 325–332.

[4] Benchohra, M.; Ntouyas, S. K. Nonlocal Cauchy problems for neutral functional differential and integrodifferential inclusions in Banach spaces. *J. Math. Anal. Appl.* 258 (2001), no. 2, 573–590.

[5] Benchohra, Mouffak; Henderson, Johnny; Ntouyas, S. K. Semilinear impulsive neutral functional differential inclusions in Banach spaces. Appl. Anal. 81 (2002), no. 4, 951-963

[6] Benchohra, M.; Henderson, J.; Ntouyas, S. K. Existence results for impulsive multivalued semilinear neutral functional differential inclusions in Banach spaces. *J. Math. Anal. Appl.* 263 (2001), no. 2, 763–780

[7] Benchohra, M.; Ntouyas, S. K. Neutral functional differential and integrodifferential inclusions in Banach spaces. *Boll. Unione Mat. Ital. Sez. B Artic. Ric. Mat.* (8) 4 (2001), no. 3, 767–782.

[8] Benchohra, M.; Ntouyas, S. K. Nonlocal Cauchy problems on semi-infinite intervals for neutral functional differential and integrodifferential inclusions in Banach spaces. *Math. Slovaca* 51 (2001), no. 5, 529–545.

[9] Deimling, Klaus Nonlinear functional analysis. Springer-Verlag, Berlin, 1985.

[10] Hernández, Eduardo; Henríquez, Hernán R. Existence of periodic solutions of partial neutral functional differential equations with unbounded delay. *J. Math. Anal. Appl.* 221 (1998), no. 2, 499–522.

[11] Hernández, Eduardo; Henríquez, Hernán R. Existence results for partial neutral functional differential equations with unbounded delay. *J. Math. Anal. Appl.* 221 (1998), no. 2, 452–475.

[12] R. H. Martin, Nonlinear Operators and Differential Equations in Banach Spaces, Robert E. Krieger Publ. Co., Florida, 1987.

[13] Pazy, A. Semigroups of linear operators and applications to partial differential equations. Applied Mathematical Sciences, 44. Springer-Verlag, New York-Berlin, 1983.

[14] B. N. Sadovskii, On a fixed point principle. *Funct. Anal. Appl.* 1 (1967), 74-76.

[15] Wang, Lianglong; Wang, Zhicheng Controllability of abstract neutral functional differential systems with infinite delay. Dyn. Contin. Discrete Impuls. Syst. Ser. B Appl. Algorithms 9 (2002), no. 1, 59–70.

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