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Eduardo Hernández M

Hernán R. Henríquez

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Estabilização de Sistemas de Controle Distribuídos com Retardamento não Limitado

Eduardo Hernández M.
Departamento de Matemática, ICMC Universidade de São Paulo,
Caixa Postal 668, 13560-970 São Carlos SP, Brazil. e-mail: lalohm@icmc.sc.usp.br
Hernán R. Henríquez.
Departamento de Matemática, Universidade de Santiago
Casilla 307, Correo-2, Santiago, Chile. e-mail : hhenriqu@lauca.usach.cl

Resumo

Neste artigo estudamos a estabilização assintótica de sistemas de controle com retardamento não limitado. Assumindo que o semigrupo de operadores associado à equação não controlada e sem retardamento é compacto, e que o espaço de fase é um espaço com memória amortecida, caracterizamos aqueles sistemas que podem ser estabilizados usando controle realimentado.

Stabilization of Distributed Control Systems with Unbounded Delay

Eduardo Hernández M. and Hernán R. Henríquez

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Abstract

In this paper we study the asymptotic stabilization of linear distributed parameter control systems with unbounded delay. Assuming that the semigroup of operators associated with the uncontrolled and non delayed equation is compact and that the phase space is a uniform fading memory space, we characterize those systems that can be stabilized using a feedback control. As consequence we conclude that every system of this type is stabilizable with an appr opriated finite dimensional control.

1 Introduction

This paper is devoted to study the stabilizing problem for hereditary control systems with infinite delay. It is well known that a general requirement for the design of several control strategies is the system to be asymptotically stable. For this reason the stabilization of systems is an important subject of the control theory.

The problem of stabilizing a linear invariant control system by a dynamic output feedback has a very extensive literature. At present the theory for control systems of finite dimension is well established and we refer to O'Reilly [32] and Wonham [45] for the most important part of the theory. Similarly, the problem of feedback stabilization of control systems with delays it has been discussed in many works, employing different approaches. The most of works devoted to this subject are concerned with the system

$$\begin{aligned} x'(t) &= L(x_t) + Bu(t) \\ y(t) &= L_1(x_t) \end{aligned}$$

where $x(t) \in I\!R^n$ denotes the state; the function $x_t : [-r, 0] \to I\!R^n$ defined by $x_t(\theta) = x(t+\theta)$ represents the "history" of x at t; $u(t) \in I\!R^m$ is the input at time t; $y(t) \in I\!R^p$ denotes the output variable; B is a $n \times m$ matrix and both L as L_1 are bounded linear maps defined on the space of continuous functions $C([-r, 0]; I\!R^n)$.

Some authors have studied different aspects of the problem of stabilization for a fixed time delay (see [1, 3, 10, 16, 19, 25, 27, 30, 31, 33, 35, 36, 39, 41, 42, 43, 44]) while some others have considered the problem of stabilization independent of delays ([2, 3, 4, 5, 6, 7, 9, 22, 23, 24, 26, 29, 40]). On the other hand, some of these works are concentrated on the design of asymptotic observers with point delays (commensurate or noncommensurate) while some others consider distributed delays.

In particular, the stabilization of the system

$$x'(t) = \int_{-r}^{0} d_\theta N(\theta) x(t+\theta) \; + \; Bu(t),$$

where $N(\cdot)$ is an $n \times n$ matrix valued map of bounded variation on $[-r, 0]$, was considered by Pandolfi in [33]. Specifically, defining

$$\Delta(\lambda) = \lambda I \; - \; \int_{-r}^{0} e^{\lambda\theta} d_\theta N(\theta)$$

it was proved that if $rank[\Delta(\lambda),\, B] = n,$ for $Re(\lambda) \geq 0,$ then there exists a bounded linear map $K : C([-r, 0]; I\!R^n) \to I\!R^m$ such that the system

$$x'(t) = \int_{-r}^{0} d_\theta N(\theta) x(t+\theta) \; + \; BK(x_t)$$

is stable (we refer to [13, 14] for the spectral properties of retarded functional differential equations).

Nevertheless, this class of equations does not include partial integro-differential equations which arise in the study of a number of problems such as heat conduction in materials with memory or population dynamics for spatially distributed populations ([28, 46]). For this reason, in a recent paper [18] we have studied the stabilization of control systems modelled by an abstract retarded functional differential equation (abbreviated, ARFDE) with delay $r > 0$ and states in a Banach space X. Specifically, let A be the infinitesimal generator of a strongly continuous semigroup of bounded linear operators $T(t)$ on X and $L : C([-r, 0]; X) \to X$ be a bounded linear map that can be represented as the Riemann-Stieltjes integral

$$L(\varphi) = \int_{-r}^{0} d_\theta \eta(\theta) \varphi(\theta),$$

where $\eta(\cdot)$ is an operator valued map with bounded variation. We consider those systems of type

$$x'(t) = Ax(t) \; + \; L(x_t) \; + \; Bu(t) \tag{1}$$

where $u(t) \in I\!R^m$ represent the input at time t and $B : I\!R^m \to X$ is a bounded linear maps which represent the control action. We proved that if $T(\cdot)$ is a compact semigroup and the natural extension of the rank condition already mentioned holds then the system (1) is stabilizable.

Our objective in this paper is to show that the foregoing described results can be appropriately extended to include some systems modelled by an ARFDE with infinite delay. As a model we take the linear system

$$\begin{aligned} \frac{\partial}{\partial t} w(\xi, t) &= \frac{\partial^2}{\partial \xi^2} w(\xi, t) + \int_{-\infty}^{0} d_\theta N(\theta) w(\xi, t+\theta) + \sum_{i=1}^{m} b_i(\xi) u_i(t), \quad 0 \leq \xi \leq 1, \;\; t \geq 0, \\ w(0, t) &= w(1, t) = 0, \end{aligned}$$

where the scalar function $N(\cdot)$ has bounded variation on $(-\infty, 0]$ and b_i, $i = 1, \cdots, m$, are appropriate functions.

In order to develop a general theory, we consider those linear invariant systems that can be modelled by an equation of type (1) where in this case the history x_t is a function from $(-\infty, 0]$ into X, defined as above by $x_t(\theta) = x(t+\theta)$, $-\infty < \theta \leq 0$. We assume that the histories x_t belong to some space, usually called the phase space for the equation, $\mathcal{B}$ which is defined axiomatically and $L : \mathcal{B} \to X$ is a bounded linear map.

Throughout this work we denote by $\mathcal{L}(X)$ the Banach algebra of bounded linear operators defined on X and by X^* the dual space of X. For a linear operator A with domain $D(A)$ and range $\mathcal{R}(A)$ in X, we represent by $\sigma(A)$ (resp. $\sigma_{ess}(A)$, $\sigma_p(A)$) the spectrum (respectively, essential, point spectrum) of A and by $\rho(A)$ the resolvent set of A. For each $\lambda \in \rho(A)$, the resolvent operator $R(\lambda, A)$ is defined by $R(\lambda, A) = (\lambda I - A)^{-1}$. If $D(A)$ is dense in X, then A' denotes the dual operator of A ([11]).

This paper is organized as follows. In section 2 we have collected some technical results about ARFDE with infinite delay and in section 3 we apply these properties to stabilize control systems described by an ARFDE with infinite delay.

2 Preliminaries

Throughout the rest of this paper X denotes a Banach space endowed with a norm $\|\cdot\|$ and $A : D(A) \to X$ is the infinitesimal generator of a strongly continuous semigroup of bounded linear operators $T(t)$ on X. In this section we are concerned with the abstract retarded functional differential equation with infinite delay

$$x'(t) = Ax(t) + L(x_t) + f(t), \quad t \geq 0, \tag{2}$$

where $f : [0, \infty) \to X$ is a locally integrable function. To develop a general theory we assume that $x_t \in \mathcal{B}$ and $L : \mathcal{B} \to X$ is a bounded linear map, where $\mathcal{B}$ is defined axiomatically.

We will employ an axiomatic definition of the phase space $\mathcal{B}$ introduced by Hale and Kato [12]. However, to establish the axioms of space $\mathcal{B}$ we prefer the terminology used in the book [20]. Thus, $\mathcal{B}$ will be a linear space of functions mapping $(-\infty, 0]$ into X endowed with a seminorm $\|\cdot\|_{\mathcal{B}}$. We will assume that $\mathcal{B}$ satisfies the following axioms:

(A) If $x : (-\infty, \sigma + a) \to X$, $a > 0$, is continuous on $[\sigma, \sigma + a)$ and $x_\sigma \in \mathcal{B}$, then for every t in $[\sigma, \sigma + a)$ the following conditions hold:

(i) x_t is in $\mathcal{B}$.

(ii) $\|x(t)\| \leq H \|x_t\|_{\mathcal{B}}$.

(iii) $\|x_t\|_{\mathcal{B}} \leq K(t - \sigma) \sup\{\|x(s)\| : \sigma \leq s \leq t\} + M(t - \sigma)\|x_\sigma\|_{\mathcal{B}}$, where $H \geq 0$ is a constant; $K, M : [0, \infty) \to [0, \infty)$, K is continuous and M is locally bounded and H, K and M are independent of $x(\cdot)$.

(A-1) For the function $x(\cdot)$ in (A), x_t is a $\mathcal{B}$-valued continuous function on $[\sigma, \sigma + a)$.

(B) The space $\mathcal{B}$ is complete.

Throughout this paper we always assume that $\mathcal{B}$ is a phase space.

It is follows from these axioms that the operator $S(t)$ defined by the expression

$$[S(t)\varphi](\theta) = \begin{cases} \varphi(0), & -t \leq \theta \leq 0, \\ \varphi(t+\theta), & -\infty < \theta < -t, \end{cases}$$

is a strongly continuous semigroup of linear operators on $\mathcal{B}$. Furthermore, we put $\mathcal{B}_0 = \{\varphi \in \mathcal{B} : \varphi(0) = 0\}$. It is clear that $\mathcal{B}_0$ is a closed subspace of $\mathcal{B}$. We denote by $S_0(t)$ the restriction of $S(t)$ to $\mathcal{B}_0$.

In the theory of retarded functional differential equations with unbounded delay frequently we need additional properties of the space $\mathcal{B}$ to obtain some results. Next we denote by C_{00} the space of continuous functions from $(-\infty, 0]$ into X with compact support. It is clear from the axioms of phase space that $C_{00} \subseteq \mathcal{B}$. In this work we consider the following axiom ([20]).

(C-2) If a uniformly bounded sequence $(\varphi^n)_n$ in C_{00} converges to a function φ in the compact-open topology then φ belongs to $\mathcal{B}$ and $\|\varphi^n - \varphi\|_{\mathcal{B}} \to 0$, as $n \to \infty$.

It is easy to see ([20]) that if (C-2) holds then the space $C_b((-\infty, 0]; X)$ (or in short C_b) formed by the bounded continuous functions $\varphi : (-\infty, 0] \to X$ is continuously included in $\mathcal{B}$. Thus, there exists a positive constant $\mu > 0$ such that $\|\varphi\|_{\mathcal{B}} \leq \mu \|\varphi\|_\infty$, for all $\varphi \in C_b$.

In this work we are interested in the asymptotic behavior of the solution operator associated with an homogeneous abstract retarded functional differential equation with infinite delay. For this reason we need to complete the description of $\mathcal{B}$, establishing the asymptotic behavior of semigroup $S(t)$.

For the necessary concepts related with the abstract Cauchy problem and the theory of strongly continuous semigroup of operators we refer to Engel and Nagel [8] and Pazy [34]. We only mention here a few concepts and results directly related with our developments. Let $G(t)$ be a strongly continuous semigroup defined on a Banach space X with infinitesimal generator C. We say that G is strongly stable if $G(t)x \to 0$, $t \to \infty$, for all $x \in X$ and we say that G is uniformly stable if $\|G(t)\| \to 0$, $t \to \infty$. Moreover, we employ the terminology and notations for spectral bound $s(C)$, growth bound $\omega_0(G)$ and essential growth bound $\omega_{ess}(G)$ from [8]. Consequently, in terms of these notations, G is uniformly stable if, and only if, $\omega_0(G) < 0$.

For completeness we also regard here that a strongly continuous semigroup $G(t)$ is said compact if $G(t)$ is a compact operator for all $t > 0$ and that G is said quasi-compact if there is $t_0 > 0$ and a compact operator R such that $\|G(t_0) - R\| < 1$. We collect in the following lemma a pair of essential results ([8]) for our development.

Lemma 1 *The following conditions are fulfilled.*

(i) *The semigroup G is quasi-compact if and only if $\omega_{ess}(G) < 0$;*

(ii) $\omega_0(G) = \max\{\omega_{ess}(G), s(C)\}$.

Related with these concepts we introduce the following axioms for the phase space $\mathcal{B}$.

(FMS) The space $\mathcal{B}$ is called a fading memory space if it satisfies (C-2) and S_0 is strongly stable.

(UFMS) The space $\mathcal{B}$ is called a uniform fading memory space if it satisfies (C-2) and S_0 is uniformly stable.

For the properties of phase spaces that satisfy these axioms we refer to [20]. We only emphasize here that if $\mathcal{B}$ satisfies (FMS), then we can choose the functions $K(\cdot)$ and $M(\cdot)$ in axiom (A-iii) as constants $K(\cdot) = K$ and $M(\cdot) = M$. If, in addition, $\mathcal{B}$ is (UFMS) then we can choose $M(t) = (1 + \mu H)\|S_0(t)\| \to 0,\ t \to \infty$.

In what follows we assume that $L : \mathcal{B} \to X$ is a bounded linear operator. It is well known that the equation (2) with the initial condition

$$x_0 = \varphi \in \mathcal{B} \tag{3}$$

has a unique mild solution $x = x(\cdot, \varphi, f)$. This means that $x : I\!R \to X$ is a function that verifies (3), the restriction on $[0, \infty)$ is continuous and satisfies the integral equation

$$x(t) = T(t)\varphi(0) + \int_0^t T(t-s)(L(x_s) + f(s))\, ds,\ \ t \geq 0. \tag{4}$$

Furthermore, the solution operator $V(t) : \mathcal{B} \to \mathcal{B}$ of the homogeneous equation

$$x'(t) = Ax(t) + L(x_t),\ \ t \geq 0, \tag{5}$$

which is given by

$$V(t)\varphi = x_t(\cdot, \varphi, 0) \tag{6}$$

defines a strongly continuous semigroup of operators ([17]). Henceforth, we represent by A_V its infinitesimal generator . In particular, we denote by $W(t)$ the solution operator corresponding to $L = 0$. It is clear that $W(t)$ is given by

$$[W(t)\varphi](\theta) = \begin{cases} T(t+\theta)\varphi(0), & -t \leq \theta \leq 0, \\ \varphi(t+\theta), & -\infty < \theta < -t, \end{cases}$$

Next we reserve the symbol α to denote the Kuratowski's measure of non-compactness on bounded sets in an appropriate space or the essential norm of bounded linear operators. Also, we will represent with the symbol $B_r[x]$ the closed ball centered at x and of radius $r \geq 0$.

We are in conditions to establish our first result about asymptotic behavior of the solution semigroup.

Theorem 1 *Assume that $\mathcal{B}$ satisfies (UFMS) and that $T(t)$ is a compact semigroup. Then $V(t)$ is quasi-compact.*

Proof. We define the operator $U(t) : \mathcal{B} \to \mathcal{B},\ t \geq 0,$ by

$$[U(t)\varphi](\theta) = \begin{cases} \displaystyle\int_0^{t+\theta} T(t+\theta-s)L(V(s)\varphi)\, ds, & -t \leq \theta \leq 0, \\ 0, & -\infty < \theta < -t, \end{cases}$$

It is clear from (4) that

$$V(t) = W(t) + U(t), \quad t \geq 0,$$

which implies that $U(t)$ is a bounded linear operator. Moreover, by obvious identification we can consider the range of $U(t)$ included in $C_0([-t,0];X)$, where the subindex 0 indicates the continuous functions that vanish at $-t$. Applying the Ascoli-Arzela's theorem it is not difficult to show that $U(t)$ is a compact operator onto $C_0([-t,0];X)$. Since $\mathcal{B}$ satisfies axiom (C-2) we can consider $C_0([-t,0];X)$ continuously included in $\mathcal{B}$, which implies that $U(t)$ is compact. Consequently, $\omega_{ess}(V) = \omega_{ess}(W)$ so that in order to complete the proof it only remains to estimate $\omega_{ess}(W)$.

To abbreviate our notations, for each $\varphi \in \mathcal{B}$ we denote by x the function $x(\theta) = \varphi(\theta),\ \theta \leq 0$, and $x(t) = T(t)\varphi(0),\ t \geq 0$. We fix $\sigma > 0$ and denote $y(t) = x(t),\ t \geq \sigma$, and $y(t) = x(\sigma),\ t \leq \sigma$. Moreover, we identify the elements $u \in X$ with the constant function $u(\theta) = u,\ -\infty < \theta \leq 0$, which belongs to the space $\mathcal{B}$. Applying these notations it is clear that

$$W(t)\varphi = x_t = y_t + S_0(t-\sigma)(x_\sigma - x(\sigma)), \quad t \geq \sigma.$$

Hence we obtain that

$$\alpha(W(t)) \leq \alpha(\{y_t : \|\varphi\|_{\mathcal{B}} \leq 1\}) + \alpha(S_0(t-\sigma)(W(\sigma) - e_0 W(\sigma))),$$

where $e_0 : \mathcal{B} \to \mathcal{B}$ denotes the linear map given by $e_0(\varphi) = \varphi(0)$. From Shin [37] we can estimate the first term on the right hand side as

$$\alpha(\{y_t : \|\varphi\|_{\mathcal{B}} \leq 1\}) \leq K\alpha(\{y(\cdot) : \|\varphi\|_{\mathcal{B}} \leq 1\}) + M(t-\sigma)\alpha(\{y_\sigma : \|\varphi\|_{\mathcal{B}} \leq 1\}),$$

where $y(\cdot)$ is considered as a continuous function on $[\sigma, t]$. Since T is a compact semigroup we obtain easily that $\{y(\cdot) : \|\varphi\|_{\mathcal{B}} \leq 1\}$ is a relatively compact set in $C([\sigma,t];X)$. Moreover, in view of that y_σ is the constant function defined by $y_\sigma(\theta) = y(\sigma) = T(\sigma)\varphi(0)$, from the definition of α is easy to see that

$$\alpha(\{y_t : \|\varphi\|_{\mathcal{B}} \leq 1\}) \leq \mu H \widetilde{M}_\sigma\, M(t-\sigma)\, \alpha(B_1[0]),$$

where $\widetilde{M}_\sigma = \sup_{0 \leq s \leq \sigma} \|T(s)\|$.

Collecting this estimation with our previous remarks about the function $M(\cdot)$ we infer that

$$\alpha(W(t)) \leq C_\sigma \|S_0(t-\sigma)\|,$$

where C_σ is a constant. This implies that

$$\begin{aligned} \omega_{ess}(W) &= \lim_{t\to\infty} \frac{\ln \alpha(W(t))}{t} \\ &\leq \lim_{t\to\infty} \frac{\ln \|S_0(t)\|}{t} \\ &= \omega_0(S_0) \end{aligned}$$

which completes the proof. ■

Collecting this result with Theorem V.3.7 in [8] we can establish the following property of asymptotic behavior for the solution semigroup associated to the homogeneous problem (5).

Corollary 1 *Assume that $\mathcal{B}$ satisfies (UFMS) and that $T(t)$ is a compact semigroup. Then the semigroup $V(t)$ is uniformly stable if and only if* $\sup Re\sigma_p(A_V) < 0$.

We return to the nonhomogeneous equation (2). It has been proved by Hino et al. [21] that the solution can be expressed by means of the variation of constants formula. Next we adopt the notations used in this paper. Moreover, in order to use this result, hereafter we assume that the space $\mathcal{B}$ satisfies (C-2) and that the operator L can be represented as

$$L(\varphi) = \int_{-\infty}^{0} d_\theta \eta(\theta)\varphi(\theta), \quad \varphi \in C_{00}, \tag{7}$$

where the map $\eta : (-\infty, 0] \to \mathcal{L}(\mathcal{X})$ has locally bounded variation, is left continuous on $(-\infty, 0)$ and $\eta(0) = 0$. Let $\Gamma^n : X \to \mathcal{B}$ be defined by

$$\Gamma^n(\theta)x = \begin{cases} (n\theta + 1)x, & -1/n \leq \theta \leq 0, \\ 0, & -\infty < \theta < -1/n, \end{cases}$$

If f is a continuous function, the formula ([21])

$$x_t(\cdot, \varphi, f) = V(t)\varphi + \lim_{n\to\infty} \int_0^t V(t-s)\Gamma^n f(s)\, ds \tag{8}$$

gives the solution of (2)-(3) in the phase space. In addition, from the proof of Theorem 3.1 in [21], specifically from the Lemma 3.3 in that work, it follows that the convergence in (8) is uniform at f, for f included in a bounded set in $C([0,t];X)$.

Remark 2.1 Assuming that the conditions of the Theorem 2.1 hold we have that the set $\Lambda = \{\lambda \in \sigma(A_V) : Re(\lambda) \geq 0\}$ is finite and consists of poles of $R(\cdot, A_V)$ with finite algebraic multiplicity ([8], Theorem V.3.7). Therefore, the phase space is decomposed as

$$\mathcal{B} = P_\Lambda \oplus Q_\Lambda, \tag{9}$$

where P_Λ and Q_Λ are spaces invariant under $V(t)$ and the space P_Λ is the range of the spectral projection Π^P corresponding to Λ. Consequently, P_Λ consists of the generalized eigen-vectors corresponding to the eigen-values $\lambda_i \in \Lambda$. We denote by $V^P(t)$, (respectively, $V^Q(t)$) the restriction of $V(t)$ on P_Λ (respectively, on Q_Λ). Similarly, A_V^P and A_V^Q represent the restrictions of A_V on P_Λ and Q_Λ, respectively. Since P_Λ is a space of finite dimension d, the semigroup $V^P(t)$ is uniformly continuous and A_V^P is a bounded linear operator defined on P_Λ. Let $\varphi^1, \varphi^2, \cdots, \varphi^d$ be a basis of P_Λ. We put $\Phi = (\varphi^1, \varphi^2, \cdots, \varphi^d)$. Consequently, proceeding as usual in the theory of functional differential equations ([14]), there is a $d \times d$ matrix G such that $\Phi(\theta) = \Phi(0)e^{G\theta}$, for $\theta \leq 0$, $V^P(t)\Phi = \Phi e^{Gt}$, $t \geq 0$, and $\sigma_p(G) = \Lambda$. Let Ψ be the dual basis of Φ associated to the decomposition (9), as constructed in [21]. In these conditions, it has been established in [21], Proposition 4.2, that there exists $x^* = col(x_1^*, x_2^*, \cdots, x_d^*) \in X^{*d}$ such that the projection $x_t^P = \Pi^P x_t$ on P_Λ of the solution of (2)-(3) is given by $x_t^P = \Phi z(t)$, where the d-vector $z(t)$ satisfies the ordinary differential equation

$$z'(t) = Gz(t) + \langle x^*, f(t) \rangle \tag{10}$$

Next we establish a direct relation between this vector x^* with the functional equation (2).

To establish our results we need some additional notations. First we observe that for every $\lambda \in \mathbb{C}$ with $Re(\lambda) \geq 0$ and for every $x \in X$, the function $e^{\lambda\theta}x$ is bounded on $(-\infty, 0]$ and, consequently, it is included in $\mathcal{B}$. This allows us to define $L_\lambda : X \to X$ and $\Delta(\lambda) : D(A) \to X$ as

$$\begin{aligned} L_\lambda x &= L(e^{\lambda\theta}x), \;\; x \in X, & (11)\\ \Delta(\lambda)x &= \lambda x \;-\; Ax \;-\; L_\lambda x, \;\; x \in D(A). & (12) \end{aligned}$$

and it is clear that L_λ is a bounded linear operator. The following spectral property is well known (see for example [38] and the references cited therein). We include here for completeness.

Lemma 2 *Assume that $\mathcal{B}$ satisfies (C-2) and let $\lambda \in \mathbb{C}$, $Re(\lambda) \geq 0$. Then $\lambda \in \sigma_p(A_V)$ if and only if there is $u \in D(A)$, $u \neq 0$, such that $\Delta(\lambda)u = 0$. In this case the function $\varphi = e^{\lambda\theta}u$ is the eigen-vector of A_V corresponding to λ.*

We also need a strengthen version of axiom (C-2).

Lemma 3 *Assume that $\mathcal{B}$ satisfies (UFMS). Let φ^n, $\varphi \in C_b$, $n \in \mathbb{N}$, be such that the sequence $(\varphi^n)_n$ is uniformly bounded and converges to φ in the compact-open topology. Then $(\varphi^n)_n$ converges to φ in the space $\mathcal{B}$.*

Proof. Without loss of generality we can assume that $\varphi = 0$. We fix $\sigma > 0$. Since $\psi^n = \varphi^n_{-\sigma} \in \mathcal{B}$, from the axioms of phase space we can write that

$$\begin{aligned} \|\varphi^n\|_{\mathcal{B}} &\leq K \max_{-\sigma\leq\theta\leq 0} \|\varphi^n(\theta)\| \;+\; M(\sigma)\|\psi^n\|_{\mathcal{B}} \\ &\leq K \max_{-\sigma\leq\theta\leq 0} \|\varphi^n(\theta)\| \;+\; \mu M(\sigma) \sup_{-\infty<\theta\leq-\sigma} \|\varphi^n(\theta)\| \end{aligned}$$

and selecting σ large enough we conclude that $\|\varphi^n\|_{\mathcal{B}} \to 0$, $n \to \infty$. ∎

In the final results of this section we assume that hypotheses of Theorem 2.1 hold and we utilize the notations introduced in Remark 2.1.

Theorem 2 *The components $x_i^* \in D(A')$, $i = 1, \cdots, d$. Furthermore, if $v \in \mathbb{R}^{d*}$ is a left eigen-vector of G corresponding to $\lambda \in \Lambda$, then $(\lambda I - A' - L'_\lambda)vx^* = 0$.*

Proof. To prove the first assertion we choose $y \in D(A)$ and we take the function $f(t) = -Ay - Ly$. We consider the equation (2) with initial condition $x_0 = y$. Since

$$x'(t) = A(x(t) - y) \;+\; L(x_t - y)$$

the solution $x(\cdot)$ satisfies $x_t - y = V(t)(0)$ so that $x_t = y$, $t \geq 0$. On the other hand, applying (10) we obtain that

$$\Pi^P x_t = \Phi < \Psi, y >= \Phi z(t),$$

where

$$z'(t) = Gz(t) + \; < x^*, -Ay - L(y) >= 0,$$

which implies that
$$< x^*, Ay + L(y) >= G < \Psi, y >$$
is a continuous function of y. This shows the assertion.

To establish the second assertion we proceed in a similar way. In this case we define $f(t) = e^{\lambda t}(\lambda y - Ay - L_\lambda y)$. Let $x(t) = e^{\lambda t}y,\ t \in I\!R$. It is clear that $x_t \in \mathcal{B}$ and $x_t = e^{\lambda t}\psi$, where $\psi(\theta) = e^{\lambda\theta}y$. Hence follows that
$$L(x_t) = e^{\lambda t}L(\psi) = e^{\lambda t}L_\lambda(y).$$
From this follows that $x(\cdot)$ is the solution of the equation (2) with initial condition $x_0 = e^{\lambda\theta}y$. Hence we obtain that
$$\begin{aligned} \Pi^P x_t &= \Phi < \Psi, x_t > \\ &= \Phi < \Psi, e^{\lambda t}e^{\lambda\theta}y > \\ &= e^{\lambda t}\Phi < \Psi, e^{\lambda\theta}y > . \end{aligned} \tag{13}$$
On the other hand, turning to use (10), the projection $\Pi^P x_t = \Phi z(t)$, where
$$z(t) = e^{Gt}z(0) + \int_0^t e^{G(t-s)} < x^*, f(s) > ds$$
which implies that
$$\begin{aligned} vz(t) &= e^{\lambda t}vz(0) + \int_0^t e^{\lambda(t-s)}v < x^*, f(s) > ds \\ &= e^{\lambda t}vz(0) + \int_0^t e^{\lambda(t-s)} < vx^*, f(s) > ds \\ &= e^{\lambda t}vz(0) + \int_0^t e^{\lambda(t-s)} < vx^*, e^{\lambda s}(\lambda y - Ay - L_\lambda y) > ds \\ &= e^{\lambda t}(vz(0) + t < vx^*, \lambda y - Ay - L_\lambda y >). \end{aligned}$$
Since from (13) we have that $e^{-\lambda t}z(t)$ is bounded on $t \geq 0$ this equality implies that $< vx^*, \lambda y - Ay - L_\lambda y >= 0$. Since y was chosen arbitrarily in $D(A)$ and $D(A)$ is dense in X, this completes the proof. ■

Proposition 1 *Let $v \in I\!R^{d*}$ be a left eigen-vector of G corresponding to $\lambda \in \Lambda$ such that $vx^* = 0$. Then $v = 0$.*

Proof. Let $\varphi \in P_\Lambda,\ \varphi \neq 0$, be an eigen-vector of A_V corresponding to λ. From Lemma 2.2 we obtain that $\varphi = e^{\lambda\theta}\varphi(0)$. For a fixed γ with $Re(\gamma + \lambda) \geq 0$ we have that $\varphi^1(\theta) = e^{\gamma\theta}\varphi(\theta) = e^{(\gamma+\lambda)\theta}\varphi(0)$ is bounded on $(-\infty, 0]$ so that $\varphi^1 \in \mathcal{B}$. We set $x(t) = e^{(\gamma+\lambda)t}\varphi(0),\ t \in I\!R$. Clearly $x_t = e^{(\gamma+\lambda)t}\varphi^1$ for $t \geq 0$. This implies that $x(\cdot)$ is a solution of equation (2) for $f(t) = e^{(\gamma+\lambda)t}[(\gamma+\lambda)\varphi(0) - A\varphi(0) - L(\varphi^1)]$. Let $z(\cdot)$ be such that $x_t^P = \Phi z(t) = \Phi < \Psi, x_t >$. From (10) we have that
$$\begin{aligned} vz'(t) &= vGz(t) + v < x^*, f(t) > \\ &= \lambda vz(t), \end{aligned}$$
which implies that $vz(t) = e^{\lambda t}c$. On the other hand, since
$$v < \Psi, x_t >= v < \Psi, e^{(\gamma+\lambda)t}\varphi^1 >= e^{\lambda t}c,\ \ t \geq 0,$$
it follows that $v < \Psi, \varphi^1 > e^{\gamma t} = c$. If we assume that $Re(\gamma) > 0$ we conclude that $v < \Psi, \varphi^1 >= 0$. In view of Lemma 2.3 we know that $\varphi^1 = e^{\gamma\theta}\varphi$ converges to φ in the space $\mathcal{B}$ as $\gamma \to 0$. This implies that $v < \Psi, \varphi >= 0$, from which we obtain $v = 0$. ■

3 Stabilization results

In this section we apply the properties studied in section 2 to the problem of stabilizing the system

$$x'(t) = Ax(t) + L(x_t) + Bu(t) \tag{14}$$

where $u(t) \in I\!R^m$ denotes the input variable and $B : I\!R^m \to X$ is a linear map and both A as L satisfy all conditions considered in section 2. Specifically, in this section we assume that the semigroup $T(t)$ is compact, the phase space satisfies (C-2) and that L can be represented in the form (7). Moreover, we keep the notations introduced in Remark 2.1 in relation with the homogeneous equation (5). In particular, the set $\Lambda := \{\lambda \in \sigma_p(A_V) : Re(\lambda) \geq 0\}$. Furthermore, since $Bu = \sum_{j=1}^{m} u_j b_j$, for some $b_j \in X$ we introduce the notation x^*B for the $d \times m$ matrix whose (i,j) coefficient is $< x_i^*, b_j >$, $i = 1, \cdots, d$, $j = 1, \cdots, m$.

We begin by establishing our concept of stabilization.

Definition 1 *The system (14) is said stabilizable if there exists a bounded linear map $F : \mathcal{B} \to I\!R^m$ such that the system*

$$x'(t) = Ax(t) + (L + BF)(x_t) \tag{15}$$

is uniformly stable.

Next we denote by $\tilde{V}(t)$ the solution semigroup associated to (15), with infinitesimal generator $\tilde{A}_V$.

Initially we establish some necessary conditions in order the system (14) to be stabilizable (we also refer the reader to [38]).

Proposition 2 *If the system (14) is stabilizable then the semigroup $V(t)$ is quasi-compact and $\mathcal{B}$ satisfies (UFMS).*

Proof. From the variation of constants formula (8) we can write

$$\tilde{V}(t)\varphi = V(t)\varphi + \lim_{n\to\infty} \int_0^t V(t-s)\Gamma^n BF\tilde{V}(s)\varphi\, ds. \tag{16}$$

Since B is a compact operator is easy to see that the operator $R_n(t)$ given by

$$R_n(t)\varphi = \int_0^t V(t-s)\Gamma^n BF\tilde{V}(s)\varphi\, ds$$

also is compact. Let $R(t) = \tilde{V}(t) - V(t)$. As was mentioned in Section 2, the convergence in (16) is uniform for $\|\varphi\|_{\mathcal{B}} \leq 1$. Consequently, $R(t)$ is a compact operator and since $\|\tilde{V}(t)\| < 1$, for t large enough it follows that V is quasi-compact.

On the other hand, proceeding as in the proof of Theorem 2.1 we can write $V(t) = W(t) + U(t)$, where $U(t)$ is a compact operator. This shows that $W(t)$ is also a quasi-compact semigroup. Since the restriction of $W(t)$ on $\mathcal{B}_0$ coincides with $S_0(t)$ we obtain that $S_0(t)$ also it is. Hence it follows that $\omega_{ess}(S_0) < 0$. Furthermore, from the spectral mapping theorem for the point spectrum ([8]) it follows that $\sigma_p(S_0(t))$ is an empty set and applying the spectral theorem for quasi-compact semigroups ([8], Theorem V.3.7) we deduce that $\omega_0(S_0) < 0$. ∎

For this reason in what follows we assume that $\mathcal{B}$ satisfies the axiom (UFMS), which in turn implies that V is a quasi-compact semigroup.

Proposition 3 *Assume that the space $\mathcal{B}$ satisfies (UFMS) and let $\lambda \in \Lambda$ be such that $\mathcal{R}[\Delta(\lambda), B] = X$. Then $rank[\lambda I - G, x^* B] = d$.*

Proof. Let $v \in \mathbb{R}^{d^*}$ be a vector such that $vG = \lambda v$ and $vx^*B = 0$. From Theorem 2.2 it follows that $\Delta'(\lambda)vx^* = 0$. Consequently, $vx^* \in \mathcal{R}[\Delta(\lambda), B]^{\perp}$ which implies that $vx^* = 0$. Applying now Proposition 2.1 we infer that $v = 0$. ∎

Now we are in conditions to establish the following result.

Theorem 3 *The system (14) is stabilizable if and only if $\mathcal{R}[\Delta(\lambda), B] = X$ for all $\lambda \in \Lambda$.*

Proof. Let $x(\cdot)$ be the mild solution of (14) with initial condition $x_0 = \varphi$. From the variation of constants formula (10) in P_Λ we can write $x_t^P = \Phi z(t)$, $t \geq 0$, where $z(t)$ satisfies

$$z'(t) = Gz(t) + x^* Bu(t) \tag{17}$$

which is a control system with states in $\mathbb{R}^d$.

Assuming first that $\mathcal{R}[\Delta(\lambda), B] = X$, for $\lambda \in \Lambda$. From the preceding proposition we obtain that $rank[\lambda I - G, x^* B] = d$ for all $\lambda \in \Lambda$. Since $\sigma_p(G) = \Lambda$ the Hautus criteria ([15]) implies that (17) is stabilizable. Consequently, there exists a $m \times d$ matrix H such that $\sigma(G + x^* BH) = \{\gamma_1, \cdots, \gamma_l\}$ is included in $\mathbb{C}^- \setminus \sigma(A_V)$.

We define $F(\varphi) = H < \Psi, \varphi >$, $\varphi \in \mathcal{B}$.

Let $y(\cdot)$ and $\tilde{x}(\cdot)$ be the mild solutions of (5) and (15), respectively, with initial condition φ.

Initially we will prove that if $\varphi \in Q_\Lambda$ then $y(t) = \tilde{x}(t)$ for all $t \geq 0$. In fact, $y_t = V(t)\varphi \in Q_\Lambda$ and $F(y_t) = H < \Psi, y_t >= 0$, which implies that $y(t)$ is also solution of (15). We conclude from this property that $\tilde{V}(t)\varphi = V(t)\varphi$. In particular, the space Q_Λ is also invariant under the operator $\tilde{V}(t)$. Consequently, with respect to the decomposition $\mathcal{B} = P_\Lambda \oplus Q_\Lambda$ we can represent $\tilde{V}(t)$ and $\tilde{A}_V$ as the block triangular operators

$$\tilde{V}(t) = \begin{bmatrix} \tilde{V}_{11}(t) & 0 \\ \tilde{V}_{21}(t) & V^Q(t) \end{bmatrix} \quad \text{and} \quad \tilde{A}_V = \begin{bmatrix} \tilde{A}_{11} & 0 \\ \tilde{A}_{21} & A^Q \end{bmatrix}. \tag{18}$$

As first application of this representation we will show that γ_i, $i = 1, \cdots, l$, are eigenvalues of $\tilde{A}_V$. In fact, let γ denote any of them. Since γ is an eigenvalue of the matrix $G + x^* BH$, then there exists $0 \neq w \in \mathbb{R}^d$ such that $\gamma w = (G + x^* BH)w$. It is clear that the function $\varphi = \Phi w \in P_\Lambda$. From (10) we have that the projection on P_Λ of the solution of (15) with initial condition φ it can be expressed as $\tilde{x}_t^P = \Phi z(t)$, where $z(t)$ satisfies

$$\begin{aligned} z'(t) &= Gz(t) + x^* BF(\tilde{x}_t) \\ &= Gz(t) + x^* BH < \Psi, \tilde{x}_t > \\ &= Gz(t) + x^* BHz(t), \end{aligned}$$

with $z(0) = w$, which implies that $z(t) = e^{\gamma t} w$ and, substituting in the expression for $\tilde{x}_t^P$, we obtain that $\tilde{x}_t^P = e^{\gamma t}\Phi w = e^{\gamma t}\varphi$. This shows that $e^{\gamma t}$ is an eigenvalue of $\tilde{V}_{11}(t)$ and that γ is an eigenvalue of $\widetilde{A}_{11}$. Since $\gamma \notin \sigma(A^Q)$, this implies that γ is an eigenvalue of $\widetilde{A}_V$.

On the other hand, from the spectral theory ([8], Proposition IV.1.16) we know that the set $\{\lambda \in \sigma(A_V) : Re(\lambda) < 0\} = \sigma(A_V^Q)$. This implies that $\{\lambda \in \sigma(A_V) : Re(\lambda) < 0\}$ is included in $\sigma(\widetilde{A}_V)$.

Next we show that $\sigma(\widetilde{A}_V)$ has no other elements. From (18) is sufficient to show that $\widetilde{A}_V$ has no other eigenvalues. Hence, in order to prove this assertion we assume that γ is an eigenvalue of $\widetilde{A}_V$. Let $\varphi \neq 0$ be the eigenvector of $\widetilde{A}_V$ corresponding to γ. Thus $\tilde{x}_t = \widetilde{V}(t)\varphi = e^{\gamma t}\varphi$. Let $\varphi^P = \Phi w$, $w \in I\!R^d$, and φ^Q be the projections of φ on P_Λ and Q_Λ, respectively. If $w = 0$ then $\varphi^P = 0$ which implies that $\varphi \in Q_\Lambda$ so that $V^Q(t)\varphi = \widetilde{V}(t)\varphi = e^{\gamma t}\varphi$ and, turning to apply the theory of semigroups we obtain that $\gamma \in \sigma_p(A_V^Q) = \{\lambda \in \sigma_p(A_V) : Re(\lambda) < 0\}$. Assuming now that $w \neq 0$ and applying (18) we infer that $\tilde{x}_t^P = \widetilde{V}_{11}(t)\varphi^P = e^{\gamma t}\varphi^P$ and, writing $\tilde{x}_t^P = \Phi z(t)$ it follows that $z(t) = e^{\gamma t} w$. Furthermore, in view of that $z(t)$ satisfies the equation

$$z'(t) = Gz(t) \; + \; x^* BH < \Psi, \tilde{x}_t >$$

it is clear that

$$\gamma w = (G \; + \; x^* BH) w$$

so that γ is an eigenvalue of $G \; + \; x^* BH$.

Finally, since the semigroup $\widetilde{V}$ is quasi-compact from Corollary 2.1 we can assert that it is uniformly stable, which shows that system (14) is stabilizable.

Conversely, if (14) is stabilizable there is a feedback control $u(t) = F(x_t)$ for which the solution semigroup $\widetilde{V}$ of (15) is uniformly stable. Let $\lambda \in \Lambda$. From Lemma 2.2 we know that $\lambda \notin \sigma_p(A + L_\lambda + BF_\lambda)$. Since $A + L_\lambda + BF_\lambda$ is an operator with compact resolvent it follows that $\lambda \in \rho(A + L_\lambda + BF_\lambda)$ which clearly implies that $\mathcal{R}[\Delta(\lambda), B] = X$. ■

The feedback control constructed in the proof of the previous theorem utilizes the history x_t. In other words, to stabilize the system (14) we require information about the complete past $x(s)$, $-\infty < s \leq t$, of the state x at time t. Certainly, this result is not applicable to realistic situations. For this reason, next we show that we also can stabilize the system (14) using only a feedback control with finite delay. In order to establish this property as a perturbation result for the stable system (15) we assume that there exists a sequence of bounded linear operators $\Pi_k : \mathcal{B} \to \mathcal{B}$ such that the support of the values $\pi_k(\varphi)$ is included in a bounded set, which is independent of $\varphi \in \mathcal{B}$, and $\|I - \Pi_k\| \to 0$, $k \to \infty$. In this case we say that $(\Pi_k)_k$ is an approximation scheme for $\mathcal{B}$. It is clear that this assumption is fulfilled if, for instance, the functions $\Pi_k\varphi$ defined by

$$\Pi_k(\varphi) = \begin{cases} \varphi(\theta), & -k \leq \theta \leq 0, \\ (\theta + k + 1)\varphi(-k), & -k - 1 \leq \theta < -k, \\ 0, & \theta < -k - 1, \end{cases}$$

belong to $\mathcal{B}$ for all $\varphi \in \mathcal{B}$ and the phase space $\mathcal{B}$ satisfies (UFMS).

Next we denote by F the operator constructed in the first part of proof of the Theorem 3.1

Corollary 2 *Assume that the hypotheses of Theorem 3.1 are fulfilled and that there exist an approximation scheme* $(\Pi_k)_k$ *for* $\mathcal{B}$*. Then there is* $k \in \mathbb{N}$ *such that the feedback control law* $u(t) = F(\Pi_k x_t)$ *stabilizes the system (14).*

Proof. With the control $u(t)$ defined as in the statement the system (14) gives

$$\begin{aligned} x'(t) &= Ax(t) + (L + BF\Pi_k)(x_t) \\ &= Ax(t) + (L + BF)(x_t) - BFR_k(x_t), \end{aligned}$$

where $R_k = I - \Pi_k$, and the solution of this equation is obtained applying the variation of constants formula

$$x_t = \widetilde{V}(t)\varphi - \lim_{n\to\infty} \int_0^t \widetilde{V}(t-s)\Gamma^n BFR_k(x_s)\, ds.$$

Since the semigroup $\widetilde{V}$ is uniformly stable, there are constants $\widetilde{N}, \gamma > 0$ such that $\|\widetilde{V}(t)\| \leq \widetilde{N}e^{-\gamma t}$, $t \geq 0$. From this it follows that

$$\|x_t\|_{\mathcal{B}} \leq \widetilde{N}e^{-\gamma t}\|\varphi\|_{\mathcal{B}} + \int_0^t \widetilde{N}e^{-\gamma(t-s)}\mu\|B\|\|F\|\|R_k\|\|x_s\|_{\mathcal{B}}\, ds$$

and the Gronwall's lemma implies that

$$\|x_t\|_{\mathcal{B}} \leq \widetilde{N}\|\varphi\|_{\mathcal{B}}\, e^{(\mu\widetilde{N}\|B\|\|F\|\|R_k\|-\gamma)t},$$

so that selecting k large enough we obtain the assertion. ■

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Eduardo Hernández M.
Departamento de Matemática
Instituto de Ciências Matemáticas e de Computação
Universidade de São Paulo - Campus de São Carlos
Caixa Postal 668
13560-970 São Carlos, SP. Brazil
E-mail : lalohm@icmc.sc.usp.br

Hernán R. Henríquez
Universidad de Santiago,
Departamento de Matemática,
Casilla 307, Correo 2,
Santiago, Chile.
e-mail : hhenriqu@lauca.usach.cl
Tel. 56-2-6812900
Fax. 56-2-6813125

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