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Some Comments on: Existence of Solutions of Abstract Nonlinear Second-order Neutral Functional Integrodifferential Equations

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Abstract

We establish the existence of mild solutions for a class of abstract second-order partial neutral functional integro-differential equations with finite delay in a Banach space using the theory of cosine families of bounded linear operators and Schaefer's fixed point theorem.

1 Introduction

The purpose of this paper is to establish the existence of mild solutions for a class of abstract second-order partial neutral functional integro-differential equations with finite delay of the abstract form

$$\begin{aligned} \frac{d}{dt}\left[x'(t)-g(t,x_t)\right] &= Ax(t)+\int_0^t F\Big(t,s,x_s,x'(s),\int_0^s f(s,\tau,x_\tau,x'(\tau))d\tau\Big)ds, \\ & \qquad\qquad t\in I=[0,a], && (1)\\ x(0) &= \varphi\in\mathcal{B}, && (2)\\ x'(0) &= z\in X, && (3)\end{aligned}$$

in a Banach space X. Here, A is the infinitesimal generator of a strongly continuous cosine family of bounded linear operators $(C(t))_{t\in\mathbb{R}}$ on X; the history $x_t:[-r,0]\to X$, $\quad x_t(\theta)=x(t+\theta)$, belongs to some abstract phase space $\mathcal{B}$ defined axiomatically, and F, f, and g are appropriate functions.

The initial-value problem (1)-(3) was addressed in [1], but the proof of the existence result contains a significant limitation. Precisely, the authors impose the condition that the cosine function $(C(t))_{t\in\mathbb{R}}$ is such that $C(t)$ is compact for $t>0$. In such case, it follows from Travis and Webb [2, p.557] that the underlying space must be finite dimensional, thereby severely weakening the applicability of the existence result Theorem 3.1 to the case of ordinary differential equations. As such, the example provided in [1] cannot be recovered as a special case. Motivated by this remark, in this paper we establish the existence of a mild solution to (1)-(3) without assuming the compactness of the operators $C(t)$.

Neutral differential equations arise in many areas of applied mathematics and for this reason, this type of equation has received much attention in recent years. The literature regarding first and second-order ordinary neutral functional differential equations is very extensive. We refer the reader to Hale & Lunel [3], Lakshmikantham, Wen & Zhang [4], Kolmanovskii & Myshkis [5], and the references therein. Furthermore, first-order partial neutral functional differential equations are studied in different works, see for example Adimy [6, 7, 8, 9], Datko [10], Hale [11], Wu [12, 13], Hernández & Henríquez [14, 15] and Hernandez [16].

More recently, some abstract second-order neutral functional differential similar to (1)-(3) have been considered in the literature, see [1, 17, 18]. However, the results in these papers possess the same technical limitation observed in [1].

The current manuscript has four sections. In Section 2 we mention notation and a few results regarding cosine function theory and phase spaces needed to establish our results. We discuss the existence of mild solutions for the neutral system (1)-(3) in Section 3. Finally, Section 4 is reserved for the discussion of some examples.

2 Preliminaries

In this section, we review some fundamental facts needed to establish our results. Regarding the theory of cosine functions of operators we refer the reader to [2, 19, 20]. Next, we only mention a few concepts and properties related to second-order abstract Cauchy problems. Throughout this paper, A is the infinitesimal generator of a strongly continuous cosine function, $(C(t))_{t\in\mathbb{R}}$, of bounded linear operators on a Banach space X. We denote by $(S(t))_{t\in\mathbb{R}}$ the sine function associated to $(C(t))_{t\in\mathbb{R}}$ which is defined by

$$S(t)x = \int_0^t C(s)x ds, \ x \in X, \ t \in \mathbb{R}.$$

Moreover, we denote by N and $\tilde{N}$ a pair of positive constants such that $\|C(t)\| \leq N$ and $\|S(t)\| \leq \tilde{N}$, for every $t \in I$.

In this paper, $[D(A)]$ represents the domain of operator A endowed with the graph norm $\| x \|_A = \| x \| + \| Ax \|$, $x \in D(A)$. The notation E stands for the space formed by the vectors $x \in X$ for which $C(\cdot)x$ is of class C^1 on $\mathbb{R}$. We know from Kisińsky [21] that E, endowed with the norm

$$\|x\|_E = \|x\| + \sup_{0\leq t\leq 1} \|AS(t)x\|, \ x \in E, \tag{4}$$

is a Banach space. The operator-valued function $\mathcal{H}(t) \doteq \begin{bmatrix} C(t) & S(t) \\ AS(t) & C(t) \end{bmatrix}$ is a strongly continuous group of bounded linear operators on the space $E \times X$ generated by the operator $\mathcal{A} = \begin{bmatrix} 0 & I \\ A & 0 \end{bmatrix}$ defined on $D(A) \times E$. It follows that $AS(t) : E \to X$ is a bounded linear operator and that $AS(t)x \to 0$ as $t \to 0$, for each $x \in E$. Furthermore, if $x : [0, \infty) \to X$ is locally integrable, then $y(t) = \int_0^t S(t-s)x(s)ds$ defines an E-valued continuous function, which is a consequence of the fact that

$$\int_0^t \mathcal{H}(t-s) \begin{bmatrix} 0 \\ x(s) \end{bmatrix} ds = \begin{bmatrix} \int_0^t S(t-s)x(s)\, ds \\ \int_0^t C(t-s)x(s)\, ds \end{bmatrix}$$

defines an $E \times X$-valued continuous function.

The existence of solutions of the second-order abstract Cauchy problem

$$\begin{aligned} x''(t) &= Ax(t) + h(t), \ t \in I, \\ x(0) &= \varsigma_0, \\ x'(0) &= \varsigma_1, \end{aligned}$$

where $h : I \to X$ is an integrable function, has been discussed in [2]. Similarly, the existence of solutions of semilinear second-order abstract Cauchy problems has been treated in [20]. We only mention here that the function $x(\cdot)$ given by

$$x(t) = C(t)\varsigma_0 + S(t)\varsigma_1 + \int_0^t S(t-s)h(s)ds, \ t \in I, \tag{5}$$

is called a mild solution of (5)-(5), and that when $\varsigma_1 \in E, x(\cdot)$ is continuously differentiable and

$$x'(t) = AS(t)\varsigma_0 + C(t)\varsigma_1 + \int_0^t C(t-s)h(s)ds, \ t \in I. \tag{6}$$

For additional material related to the properties and application of cosine function theory, we refer the reader to [2, 19, 20].

In this work, we employ an axiomatic definition of the phase space $\mathcal{B}$ similar to the one used in [22]. Specifically, here $\mathcal{B}$ stands for a linear space of functions mapping $[-r, 0]$ into X endowed with a seminorm $\|\cdot\|_{\mathcal{B}}$ satisfying the following axioms:

(A) If $x : [-r, \sigma + b) \to X$, $b > 0$, is continuous on $[\sigma, \sigma + b)$ and $x_\sigma \in \mathcal{B}$, then for every $t \in [\sigma, \sigma + b)$, the following conditions hold:

(i) x_t is in $\mathcal{B}$,

(ii) $\| x(t) \| \leq H \| x_t \|_{\mathcal{B}}$,

(iii) $\| x_t \|_{\mathcal{B}} \leq K(t-\sigma) \sup\{\| x(s) \| : \sigma \leq s \leq t\} + M(t-\sigma) \| x_\sigma \|_{\mathcal{B}}$,

where $H > 0$ is a constant; $K, M : [0, \infty) \to [1, \infty)$, $K(\cdot)$ is continuous, $M(\cdot)$ is locally bounded and H, K, M are independent of $x(\cdot)$.

(A-1) For the function $x(\cdot)$ in **(A)**, x_t is a $\mathcal{B}$-valued continuous function on $[\sigma, \sigma + b)$.

(B) The space $\mathcal{B}$ is complete.

The terminologies and notations used in this paper coincide with those generally used in functional analysis. In particular, if $(Z, \|\cdot\|_Z)$ and $(Y, \|\cdot\|_Y)$ are Banach spaces, we indicate by $\mathcal{L}(Z; Y)$ the Banach space of bounded linear operators from Z into Y endowed with the uniform operator topology; we abbreviate this notation to $\mathcal{L}(Z)$ whenever $Z = Y$. Next, $C(I; Z)$ is the space of continuous functions from I into Z endowed with the norm of uniform convergence and $B_r(x : Z)$ denotes the closed ball with center at x of radius r in Z. Additionally, for a bounded function $\xi : I \to Z$ and $0 \le t \le a$, we will employ the notation $\xi_{Z,t}$ to mean

$$\xi_{Z,t} = \sup \{\|\xi(s)\|_Z : s \in [0, t]\}, \tag{7}$$

and we will write more simply ξ_t when no confusion about the space Z arises.

To conclude this section, we recall the following well-known result (due to Schaeffer) for convenience.

Theorem 1 *[23, Theorem 6.5.4] Let D be a closed convex subset of a Banach space Z and assume that $0 \in D$. Let $G : D \to D$ be a completely continuous map. Then, either the map G has a fixed point in D or the set $\{z \in D : z = \lambda G(z),\ 0 < \lambda < 1\}$ is unbounded.*

3 Main Result

We begin with the following lemma which plays a crucial role in establishing the compactness of the solution operator.

Lemma 1 *Let $(Z_i, \|\cdot\|_i)$, $i = 1, 2, 3$, be Banach spaces, $L : J \times Z_1 \to Z_2$, $J = [0, b]$, be a function, $\{R(t) : t \in J\} \subset \mathcal{L}(Z_2, Z_3)$, and assume that the following conditions are satisfied:*

(a) *The function $L(\cdot)$ satisfies the following conditions:*

- **(i)** *For every $r > 0$, the set $L(J \times B_r(0, Z_1))$ is relatively compact in Z_2.*
- **(ii)** *The function $L(t, \cdot) : Z_1 \to Z_2$ is continuous a.e. $t \in J$, and for each $z \in Z_1$, the function $L(\cdot, z) : J \to Z_2$ is strongly measurable.*
- **(iii)** *There exist an integrable function $m_L : J \to [0, \infty)$ and a continuous function $W_L : [0, \infty) \to (0, \infty)$ such that $\| L(t, z) \|_2 \le m_L(t) W_L(\| z \|_1)$, for every $(t, z) \in J \times Z_1$.*

(b) *The operator family $(R(t))_{t \in J}$ is strongly continuous (this means that $t \to R(t)z$ is continuous on J, for every $z \in Z_2$).*

Let $\Gamma : C(J; Z_1) \to C(J; Z_3)$ be the map defined by

$$\Gamma u(t) = \int_0^t R(t - s) L(s, u(s)).$$

Then, $\Gamma(\cdot)$ is completely continuous.

Proof. It is clear that $\Gamma(\cdot)$ is well defined, continuous and from the compactness of $L(J \times B_r(0, Z_1))$, $r > 0$, and condition (b), it follows that the set $\{ R(s)L(\theta, z) : s, \theta \in J, z \in B_r(0, Z_1) \}$ is relatively compact in Z_3. For any $u \in B_r(0; C(J; Z_1))$, it follows from the Mean Value Theorem for the Bochner integral, see [24, Lemma 2.1.3], that

$$\Gamma u(t) \in t\,\overline{co(\{ R(s)L(\theta, z) : s \in J, \theta \in J,\ z \in B_r(0, Z_1)\})}^{Z_3}$$

where $co(\cdot)$ denote the convex hull, which shows that $\{\Gamma u(t) : u \in B_r(0; C(J; Z_1))\}$ is relatively compact in Z_3, for every $t \in J$.

Now, we prove that $\Gamma(B_r(0; C(J; Z_1)) = \{\Gamma u : u \in B_r(0; C(J; Z_1))\}$ is equicontinuous on J. Let $\varepsilon > 0$ and $r > 0$. From the strong continuity of $(R(t))_{t\in J}$ and the compactness of $L(J \times B_r(0; Z_1))$, we can choose $\delta > 0$ such that

$$\| R(t)L(s, z) - R(t')L(s, z) \|_3 \leq \varepsilon, \qquad t', t, s \in J,\ z \in B_r(0, Z_1),$$

when $| t - t' | \leq \delta$. Under these conditions, for $u \in B_r(0, C(J; Z_1))$, $t \in J$ and $| h | \leq \delta$ such that $t + h \in J$, we see that

$$\begin{aligned} \| \Gamma u(t + h) - \Gamma u(t) \|_3 &\leq \int_0^t \| (R(t + h - s) - R(t - s))L(s, u(s)) \|_3\, ds \\ &\quad + \sup_{\theta\in J} \| R(\theta) \|_{\mathcal{L}(Z_2;Z_3)} \int_t^{t+h} \| L(s, u(s)) \|_2\, ds \\ &\leq \varepsilon b + \sup_{\theta\in J} \| R(\theta) \|_{\mathcal{L}(Z_2;Z_3)}\, W_L(r) \int_t^{t+h} m_L(s)ds, \end{aligned}$$

which shows the equicontinuity at $t \in J$.

The assertion is now a consequence of the Azcoli-Arzela Theorem. This completes the proof. ■

Remark 1 *If $u(\cdot)$ is a solution of (1)-(3) and $t \to g(t, u_t)$ is smooth enough, then from (5) we obtain*

$$\begin{aligned} u''(t) = C(t)\varphi(0) + S(t)z + \int_0^t S(t - s)\frac{\partial}{\partial s}g(s, u_s)ds \\ + \int_0^t S(t - s) \int_0^s F(s, \tau, x_\tau, x'(\tau), \int_0^\tau f(\tau, \theta, x_\theta, x'(\theta))d\theta)d\tau ds\,,\ t \in I, \end{aligned}$$

which implies that

$$\begin{aligned} u(t) &= C(t)\varphi(0) + S(t)\,[z - g(0, \varphi)] + \int_0^t C(t - s)g(s, u_s)ds \\ &\quad + \int_0^t S(t - s) \int_0^s F(s, \tau, x_\tau, x'(\tau), \int_0^\tau f(\tau, \theta, x_\theta, x'(\theta)d\theta)d\tau ds, \qquad t \in I. \qquad (8) \end{aligned}$$

This expression motivates the following definition.

Definition 1 *A function $u : [-r, a] \to X$ is a mild solution of (1)-(3) if $u_0 = \varphi$, $u|_I \in C(I; X)$ and (8) is satisfied.*

Remark 2 *In what follows, it is convenient to introduce the function $y : [-r, a] \to X$ defined by $y_0 = \varphi$ and $y(t) = C(t)\varphi(0) + S(t)[z - g(0, \varphi)]$, for $t \in I$. Also, we let $N_1 = \sup_{\theta \in I} \|AS(\theta)\|_{\mathcal{L}(E;X)}$.*

We study system (1)-(3) under the following assumptions:

(H1) A is the infinitesimal generator of a strongly continuous cosine family$(C(t))_{t\in\mathbb{R}}$ of bounded linear operators on X.

(H2) The function $g(\cdot)$ is E-valued, $g : I \times \mathcal{B} \to X$ is continuous, and the following conditions hold:

(1) The function $g : I \times \mathcal{B} \to E$ is completely continuous;

(2) There exist positive constants c_1, c_2, an integrable function $m_g : [0, \infty) \to [0, \infty)$, and a continuous nondecreasing function $W_g : [0, \infty) \to (0, \infty)$ such that

$$\begin{aligned} \|g(t, \psi)\|_E &\leq m_g(t) W_g (\|\psi\|_{\mathcal{B}}), & (t, \psi) \in I \times \mathcal{B}, \\ \|g(t, \psi)\| &\leq c_1 \|\psi\|_{\mathcal{B}} + c_2, & (t, \psi) \in I \times \mathcal{B}; \end{aligned}$$

(3) Let $\mathcal{F} = \{x : [-r, a] \to X : x_0 = 0 \text{ and } x|_I \in C(I; X)\}$ endowed with the norm of uniform convergence. Then, for every $r > 0$, the set of functions

$$\{t \mapsto g(t, x_t + y_t) : x \in B_r(0, \mathcal{F})\}$$

is equicontinuous on I.

(H3) The function $f : I \times I \times \mathcal{B} \times X \to X$ satisfies the following conditions:

(1) $f(t, s, \cdot, \cdot) : \mathcal{B} \times X \to X$ is continuous a.e. $(t, s) \in I \times I$;

(2) $f(\cdot, \cdot, \psi, x) : I \times I \to X$ is strongly measurable, for every $(\psi, x) \in \mathcal{B} \times X$;

(3) There exist a continuous function $m_f : [0, \infty) \times [0, \infty) \to [0, \infty)$ and a continuous nondecreasing function $W_f : [0, \infty) \to (0, \infty)$ such that

$$\|f(t, s, \psi, x)\| \leq m_f(t, s) W_f (\|\psi\|_{\mathcal{B}} + \|x\|),$$

for all $(t, s, \psi, x) \in I \times I \times \mathcal{B} \times X$.

(H4) The function $F : I \times I \times \mathcal{B} \times X \times X \to X$ satisfies the following conditions:

(1) $F(t, s, \cdot, \cdot, \cdot) : \mathcal{B} \times X \times X \to X$ is continuous a.e. $(t, s) \in I \times I$;

(2) $F(\cdot, \cdot, \psi, x, y) : I \times I \to X$ is strongly measurable, for every $(\psi, x, y) \in \mathcal{B} \times X \times X$;

(3) There exist an integrable function $m_F : [0,\infty) \times [0,\infty) \to [0,\infty)$ and a continuous nondecreasing function $W_F : [0,\infty) \to (0,\infty)$ such that

$$\|F(t,s,\psi,x,y)\| \le m_F(t,s)W_F\left(\|\psi\|_{\mathcal{B}} + \|x\| + \|y\|\right),$$

for every $(t,s,\psi,x,y) \in I \times I \times \mathcal{B} \times \mathrm{X} \times \mathrm{X}$;

(4) $F(I \times I \times B_r(0,\mathcal{B}) \times B_r(0,X) \times B_r(0,X))$ is relatively compact in X, for all $r > 0$.

Remark 3 *In comparison to the hypotheses used in [1], note that condition (H1) is assumed in both papers; the compactness of the cosine family is no longer imposed in the current manuscript; and the groups of hypotheses (H_3)-(H_4), (H_5)-(H_7), and (H_8)-(H_{11}) from [1] are replaced in the current work by **(H2)**, **(H3)**, and **(H4)**, respectively.*

We now state and prove our main result.

Theorem 2 *Assume that **(H1)-(H4)** are satisfied and $\varphi(0) \in E$. If $\mu = 1 - c_1 > 0$ and*

$$\int_0^a M^*(s)ds < \int_{\frac{(c_2+c_3)}{\mu}}^{\infty} \frac{ds}{W_g(s) + W_f(s) + W_F(s)}, \tag{9}$$

where

$$M^*(s) = \max\left\{\frac{K_aN + N_1}{\mu} m_g(s),\ \frac{\tilde{N}K_a + N}{\mu}\int_0^s m_F(s,\tau)d\tau,\ m_f(s,s)\right\} \tag{10}$$

and $c_3 = \|y_s\|_{\mathcal{B},a} + \|y'\|_a$ (cf. (7) for notation), then there exists a mild solution of (1)-(3) on I.

Proof Let $Y = C(I;X)$, $\mathcal{F}$ be defined as in **(H2)**(3) and the space $Z = \mathcal{F} \times Y$ endowed with the norm $\| (u,v) \| = \| u \|_a + \| v \|_a$. On the space Z we define the map $\Gamma : Z \to Z$ by $\Gamma(u,v) = (\Gamma_1(u,v), \Gamma_2(u,v))$ where

$$\begin{aligned}\Gamma_1(u,v)(t) &= \int_0^t C(t-s)g(s,\bar{u}_s)ds \\ &+ \int_0^t S(t-s)\int_0^s F(s,\tau,\bar{u}_\tau,\bar{v}(\tau), \int_0^\tau f(\tau,\theta,\bar{u}_\theta,\bar{v}(\theta))d\theta)d\tau ds, \qquad t \in I,\end{aligned} \tag{11}$$

$$\begin{aligned}\Gamma_2(u,v)(t) &= g(t,\bar{u}_t) + \int_0^t AS(t-s)g(s,\bar{u}_s)ds \\ &+ \int_0^t C(t-s)\int_0^s F(s,\tau,\bar{u}_\tau,\bar{v}(\tau), \int_0^\tau f(\tau,\theta,\bar{u}_\theta,\bar{v}(\theta))d\theta)d\tau ds, \qquad t \in I,\end{aligned} \tag{12}$$

where $\bar{u}_s = u_s + y_s$ and $\bar{v}(s) = v(s) + y'(s)$. The well-definedness and continuity of Γ_1 and Γ_2 follow directly from the properties of the cosine family and the associated mappings $g(\cdot), f(\cdot)$ and $F(\cdot)$. In preparation for applying Theorem 1, we first obtain a priori estimates of the solutions to the integral equation $z = \lambda\Gamma z$, $\lambda \in (0,1)$. To this end, let $(u^\lambda(\cdot), v^\lambda(\cdot))$ be a solution of $z = \lambda\Gamma z$. Using **(H2)-(H4)**, together with the axioms of the phase space $\mathcal{B}$, we obtain, for all $t \in I$,

$$\begin{aligned} \left\|u^\lambda(t)\right\| \leq\; & N \int_0^t m_g(s) W_g\left(\alpha_\lambda(s)\right) ds \\ & + \tilde{N} \int_0^t \int_0^s m_F(s,\tau) W_F\left(\alpha_\lambda(\tau) + \int_0^\tau m_f(\tau,\theta) W_f\left(\alpha_\lambda(\theta)\right) d\theta\right) d\tau ds, \end{aligned} \tag{13}$$

where

$$\alpha_\lambda(s) \;=\; K_a \parallel u^\lambda \parallel_s + \parallel y_s \parallel_{\mathcal{B},a} + \parallel v^\lambda \parallel_s + \parallel y' \parallel_a . \tag{14}$$

Similarly, we obtain that (with the additional help of **(H2)**(2)), for all $t \in I$,

$$\begin{aligned} \left\|v^\lambda(t)\right\| \leq\; & c_1\alpha_\lambda(t) + c_2 + N_1 \int_0^t m_g(s) W_g\left(\alpha_\lambda(s)\right) ds \\ & + N \int_0^t \int_0^s m_F(s,\tau) W_F(\alpha_\lambda(\tau) + \int_0^\tau m_f(\tau,\theta) W_f\left(\alpha_\lambda(\theta)\right) d\theta) d\tau ds. \end{aligned} \tag{15}$$

Using (13)-(15), along with the fact that $\mu = 1 - c_1 > 0$, yields after some simplification and a rearrangement of terms

$$\begin{aligned} \alpha_\lambda(t) \leq\; & \frac{c_2 + c_3}{\mu} + \frac{K_a N + N_1}{\mu} \int_0^t m_g(s) W_g(\alpha_\lambda(s)) ds \\ & + \frac{\tilde{N} K_a + N}{\mu} \int_0^t \int_0^s m_F(s,\tau) W_F(\alpha_\lambda(\tau) + \int_0^\tau m_f(\tau,\theta) W_f(\alpha_\lambda(\theta)) d\theta) d\tau ds. \end{aligned} \tag{16}$$

Now, add the term $\int_0^t m_f(t,s) W_f\left(\alpha_\lambda(s)\right) ds$ to both sides of (16) and let

$$\beta_\lambda(t) = \alpha_\lambda(t) + \int_0^\tau m_f(t,s) W_f\left(\alpha_\lambda(s)\right) ds. \tag{17}$$

Using (17) in (16) yields

$$\begin{aligned}\beta_\lambda(t) &\le \frac{c_2+c_3}{\mu} + \frac{NK_a+N_1}{\mu}\int_0^t m_g(s)W_g\left(\beta_\lambda(s)\right)ds \\ &+\frac{\tilde{N}K_a+N}{\mu}\int_0^t\int_0^s m_F(s,\tau)W_F\left(\beta_\lambda(\tau)\right)d\tau ds \\ &+\int_0^t m_f(t,s)W_f\left(\beta_\lambda(s)\right)ds,\ t\in I.\end{aligned} \tag{18}$$

Denote the right-hand side of (18) by $\gamma_\lambda(t)$. Computing $\gamma'_\lambda(t)$ and observing that $\beta_\lambda(t) \le \gamma_\lambda(t)$, for $t \in I$, we arrive at

$$\begin{aligned}\gamma'_\lambda(t) &\le \frac{K_aN+N_1}{\mu}m_g(t)W_g(\gamma_\lambda(t)) + \frac{NK_a+N}{\mu}\int_0^t m_F(t,s)W_F\left(\gamma_\lambda(s)\right)ds \\ &\quad +m_f(t,t)W_f\left(\gamma_\lambda(t)\right) \\ &\le M^*(t)\left[W_g\left(\gamma_\lambda(t)\right)+W_F\left(\gamma_\lambda(t)\right)+W_f\left(\gamma_\lambda(t)\right)\right], \qquad t\in I,\end{aligned} \tag{19}$$

where $M^*(t)$ is defined in (10). Integrating (19) over $(0,t)$ then yields (using (9))

$$\int_{\gamma_\lambda(0)}^{\gamma_\lambda(t)} \frac{ds}{W_g(s)+W_f(s)+W_F(s)} \le \int_0^t M^*(s)ds < \int_{\frac{(c_2+c_3)}{\mu}}^{\infty} \frac{ds}{W_g(s)+W_f(s)+W_F(s)}ds,$$

where $\gamma_\lambda(0) = \frac{c_2+c_3}{\mu}$. Consequently, both $\left\|u^\lambda\right\|_t$ and $\left\|v^\lambda\right\|_t$ are bounded independent of λ and t, so that $\left\|u^\lambda(t)\right\|$ and $\left\|v^\lambda(t)\right\|$ are as well. Hence, $\{z^\lambda = (u^\lambda, v^\lambda) : z^\lambda = \lambda\Gamma z^\lambda,\ \lambda \in (0,1)\}$ is a bounded subset of Z, as desired.

It remains to show that $\Gamma_1(\cdot)$ and $\Gamma_2(\cdot)$ are completely continuous operators. To this end, we introduce the decompositions $\Gamma_1 = \Gamma_1^1 + \Gamma_1^2$ and $\Gamma_2 = \Gamma_2^1 + \Gamma_2^2$, where

$$\begin{aligned}\Gamma_1^1(u,v)(t) &= \int_0^t C(t-s)g(s,\bar{u}_s)ds, \\ \Gamma_2^1(u,v)(t) &= g(t,\bar{u}_t) + \int_0^t AS(t-s)g(s,\bar{u}_s)ds.\end{aligned}$$

Since the operator family $(AS(t))_{t\ge0}$ is strongly continuity in E, from Lemma 1, the properties of $g(\cdot)$ and the Azcoli-Arzela theorem, we can conclude that the maps Γ_i^1, $i=1,2$, are completely continuous. To prove that the maps Γ_i^2, $i=1,2$, are completely continuous, we need to show that the map $L : I \times Z \to X$ defined by

$$L(s,u,v) = \int_0^s F(s,\tau,\bar{u}_\tau,\bar{v}(\tau),\int_0^\tau f(\tau,\theta,\bar{u}_\theta,\bar{v}(\theta))d\theta)d\tau,$$

is completely continuous. It is easy to see from the assumptions that the map $L(\cdot)$ is continuous. Next, we prove that the set $\{L(s,u,v) : s \in I, (u,v) \in B_r(0,Z)\}$ is relatively compact in X, for every $r > 0$. Let $(u,v) \in B_r(0,Z)$. From the phase space axioms and the assumptions on f and F, we infer that

$$\begin{aligned} \| u_s + y_s \| &\leq K_a r + \| y_s \|_{\mathcal{B},a} := r_1, \qquad && s \in I, \\ \| v + y' \|_a &\leq r + \| y' \|_a := r_2, && s \in I, \end{aligned}$$

and hence

$$\| \int_0^\tau f(\tau, \theta, \bar{u}_\theta, \bar{v}(\theta)) d\theta \| \leq \int_0^\tau m_f(\tau,\theta) W_f(r_1 + r_2) d\theta \leq \| m_f \|_\infty W_f(r_1 + r_2) := r_3,$$

for every $\tau \in I$. From these estimates, it follows that

$$\begin{aligned} U &= \{F(s, \tau, \bar{u}_\tau, \bar{v}(\tau), \int_0^\tau f(\tau, \theta, \bar{u}_\theta, \bar{v}(\theta)) d\theta) : (s, \tau, u, v) \in I^2 \times B_r(0,Z)\} \\ &\subset \{F(s, \tau, \psi, x, y) : s, \tau \in I, \| \psi \|_{\mathcal{B}} \leq r_1, \| x \| \leq r_2, \| y \| \leq r_3\}, \end{aligned}$$

which enables us to conclude that the set U is relatively compact in X since $F(\cdot)$ is completely continuous. Now, from the Mean Value Theorem for the Bochner integral, (see [24, Lemma 2.1.3]), we infer that $L(s,u,v) \in \overline{sco(U)}^X$, for every $(s,u,v) \in I \times B_r(0,Z)$, and hence

$$\{L(s,u,v) : (s,u,v) \in I \times B_r(0,Z)\} \subset \bigcup_{t \in I} \overline{tco(U)}^X.$$

This proves that the set $L(I \times B_r(0,Z))$ is relatively compact in X and so, L is completely continuous.

An appropriate application of Lemma 1 enables us to infer that Γ_1^2 and Γ_2^2 are completely continuous which, in turn, shows that Γ_1 and Γ_2 are also completely continuous.

Finally, we can apply Theorem 1 to conclude that $\Gamma(\cdot)$ has a fixed point $(u,v) \in Z$. Clearly, $u(\cdot)$ is differentiable on I, $u'(\cdot) = v(\cdot)$ and the function $u(\cdot) + y(\cdot)$ is a mild solution of (1)-(3), as desired. This completes the proof. ■

By using the Banach contraction mapping principle, we can also establish the existence of mild solutions for system (1)-(3).

Theorem 3 *Assume that $F(\cdot), f(\cdot)$ and $g(\cdot)$ are continuous functions and that there exist constants L_F, L_f and L_g such that*

$$\begin{aligned} \| g(t,\psi_1) - g(t,\psi_2) \|_E &\leq L_g \| \psi_1 - \psi_2 \|_{\mathcal{B}}, \\ \| f(t,s,\psi_1,x) - f(t,s,\psi_2,\bar{x}) \| &\leq L_g (\| \psi_1 - \psi_2 \|_{\mathcal{B}} + \| x - \bar{x} \|), \\ \| F(t,s,\psi_1,x,y) - F(t,s,\psi_2,\bar{x},\bar{y}) \| &\leq L_F (\| \psi_1 - \psi_2 \|_{\mathcal{B}} + \| x - \bar{x} \| + \| y - \bar{y} \|). \end{aligned}$$

If $\Theta = K_a \Big[L_g (1 + a(N + N_1)) + a^2 (\tilde{N} + N) L_F (1 + L_f a) \Big] < 1$, then there exists a unique mild solution of (1)-(3).

Proof. Let $Z = \mathcal{F} \times C(I;X)$ (as in Theorem 3.1) and define $\Gamma : Z \to Z$ by $\Gamma(u,v) = (\Gamma_1(u,v), \Gamma_2(u,v))$, where $\Gamma_1(u,v)$ and $\Gamma_2(u,v)$ are given by (3.4) and (3.5), respectively. The continuity and well-definedness of Γ follow directly from the conditions on f, g and F. It remains to show that Γ is a strict contraction on Z. To this end, let $(u_1,v_1),(u_2,v_2) \in Z$ and observe that standard computations involving the phase space axioms and growth conditions yield

$$\begin{aligned}
\|\Gamma_1(u_1,v_1) - \Gamma_1(u_2,v_2)\|_a &\leq K_a\left[aNL_g + a^2\tilde{N}L_F(1+L_f a)\right]\|\bar{u}_1 - \bar{u}_2\|_a \\
&\quad + a^2\tilde{N}L_F(1+L_f a)\|\bar{v}_1 - \bar{v}_2\|_a, \qquad (20)\\
\|\Gamma_2(u_1,v_1) - \Gamma_2(u_2,v_2)\|_a &\leq K_a\left[L_g(1+N_1 a) + a^2 NL_F(1+L_f a)\right]\|\bar{u}_1 - \bar{u}_2\|_a \\
&\quad + a^2 NL_F(1+L_f a)\|\bar{v}_1 - \bar{v}_2\|_a. \qquad (21)
\end{aligned}$$

Consequently, using (20) and (21) together yields

$$\begin{aligned}
&\|\Gamma(u_1,v_1) - \Gamma(u_2,v_2)\|_Z \\
&\quad = \|\Gamma_1(u_1,v_1) - \Gamma_1(u_2,v_2)\|_a + \|\Gamma_2(u_1,v_1) - \Gamma_2(u_2,v_2)\|_a \\
&\quad \leq K_a\left[L_g(1+a(N+N_1)) + a^2(\tilde{N}+N)L_F(1+L_f a)\right]\|\bar{u}_1 - \bar{u}_2\|_a \\
&\qquad + a^2(\tilde{N}+N)L_F(1+L_f a)\|\bar{v}_1 - \bar{v}_2\|_a \\
&\quad \leq \Theta\left[\|\bar{u}_1 - \bar{u}_2\|_a + \|\bar{v}_1 - \bar{v}_2\|_a\right].
\end{aligned}$$

Hence, we conclude that Γ is a strict contraction and hence, by the Banach contraction mapping principle, has a unique fixed point $(u,v) \in Z$. Now, as in the conclusion of the proof of Theorem 2, it is clear that $u(\cdot)$ is differentiable on I, $u'(\cdot) = v(\cdot)$, and that the function $u(\cdot) + y(\cdot)$ (cf. Remark 2) is a mild solution of (1)-(3), as desired. ∎

4 Example

In this section we apply the results established in this paper to partial differential equations. We have already introduced the required technical framework in Section 3.

Let $X = L^2([0,\pi], \|\cdot\|)$ and $A : D(A) \subset X \to X$ be defined by $Af(\xi) = f''(\xi)$, where $D(A) = \{f \in H^2(0,\pi) : f(0) = f(\pi) = 0\}$. It is well known that A is the infinitesimal generator of a strongly continuous cosine function, $(C(t))_{t\in\mathbb{R}}$, on X. Furthermore, A has a discrete spectrum and the eigenvalues are $-n^2$, $n \in I\!N$, with corresponding normalized eigenvectors $z_n(\xi) := \left(\frac{2}{\pi}\right)^{1/2}\sin(n\xi)$. The following hold:

(a) $\{z_n : n \in \mathbb{N}\}$ is an orthonormal basis of X.

(b) If $\varphi \in D(A)$, then $A\varphi = -\sum_{n=1}^{\infty} n^2 < \varphi, z_n > z_n$. It follows from this expression that $i_c : [D(A)] \to X$ is compact.

(c) For $\varphi \in X$, $C(t)\varphi = \sum_{n=1}^{\infty}\cos(nt) < \varphi, z_n > z_n$. It then follows that $S(t)\varphi = \sum_{n=1}^{\infty}\frac{\sin(nt)}{n} < \varphi, z_n > z_n$, $S(t)$ is a compact operator for $t \in \mathbb{R}$, and that $\|C(t)\| = \|S(t)\| = 1$, for all $t \in \mathbb{R}$. Additionally we observe that the operators $C(2\pi k)$, $k \in \mathbb{N}$ are not compact.

(d) If Φ is the group of translations on X defined by $\Phi(t)x(\xi) = \tilde{x}(\xi + t)$, where $\tilde{x}(\cdot)$ is the extension of $x(\cdot)$ with period 2π, then $C(t) = \frac{1}{2}(\Phi(t) + \Phi(-t))$ and $A = B^2$, where B is the generator of Φ and $E = \{x \in H^1(0,\pi) : x(0) = x(\pi) = 0\}$, (see [19] for details). In particular, we observe that $i_c : E \to X$ is compact.

To treat our example, we choose the phase space $\mathcal{B} := C([-r,0];X)$. Clearly, $\mathcal{B}$ is a phase space which satisfies the axioms **(A)**, **(A-1)**, **(B)** given in Section 2. Moreover, $H = 1$ and $K \equiv M \equiv 1$ on I.

Consider the partial neutral differential equation

$$\begin{aligned}
\frac{\partial}{\partial t}[\frac{\partial u(t,\xi)}{\partial t} &+ \int_{-r}^{0}\int_{0}^{\pi} b(\theta,\eta,\xi)u(t+\theta,\eta)d\eta d\theta] = \frac{\partial^2 u(t,\xi)}{\partial \xi^2} \\
&+ \int_0^t \left(a_1(t,s)(u(s-r,\xi) + u'(s,\xi)) + \int_0^s a_2(s,\tau)(u(\tau - r,\xi) + u'(\tau,\xi))d\tau \right) ds, \\
&\qquad t \in I = [0,a], \xi \in J = [0,\pi], \qquad (22)
\end{aligned}$$

subject to the initial conditions

$$\begin{aligned}
u(t,0) &= u(t,\pi) = 0, & t \in I, \qquad (23)\\
u_0 &= \varphi, & (24)\\
\frac{\partial u(0,\xi)}{\partial t} &= z(\xi), & \xi \in J, \qquad (25)
\end{aligned}$$

where $\varphi \in \mathcal{B}$, $z \in X$, the functions a_i, $i = 1,2$, are continuous and

(i) The functions $b(\theta,\eta,\xi)$, $\frac{\partial}{\partial \xi}b(\theta,\eta,\xi)$, $\frac{\partial^2}{\partial \xi^2}b(\theta,\eta,\xi)$ are continuous on $[-r,0] \times J^2$, $b(\theta,\eta,\pi) = b(\theta,\eta,0) = 0$, for every $(\theta,\eta) \in [-r,0] \times J$. We introduce the notation,

$$\Lambda_1 := \max\left\{ \int_0^\pi \int_{-r}^0 \int_0^\pi \left(\frac{\partial^j}{\partial \xi^j} b(\theta,\eta,\xi)\right)^2 d\eta d\theta d\xi : j = 0,1,2 \right\}.$$

By defining the functions $f : I^2 \times \mathcal{B} \times X \to X$, $g : I \times \mathcal{B} \to X$ and $F : I^2 \times \mathcal{B} \times X^2 \to X$ by

$$\begin{aligned}
g(t,\psi)(\xi) &:= \int_{-r}^0 \int_0^\pi b(\theta,\eta,\xi)\psi(\theta,\eta)d\eta d\theta, \\
F(t,s,\psi,x,y)(\xi) &:= a_1(t,s)\left(\psi(-r,\xi) + x(\xi)\right) + y(\xi), \\
f(s,\tau,\psi,x)(\xi) &:= a_2(s,\tau)\left(\psi(-r,\xi) + x(\xi)\right),
\end{aligned}$$

the neutral system (22)-(25) can be written in the form of the abstract Cauchy problem (1)-(3). It is easy to see that $F(\cdot), f(\cdot)$ are continuous operators, $F(t,s,\cdot), f(t,s,\cdot)$ are bounded linear operators, and

$$\begin{aligned}
\| F(t,s,\psi,x,y) \| &\leq \max\{\|a_1\|_\infty, 1\}\left(\| \psi \|_{\mathcal{B}} + \| x \| + \| y \|\right), \\
\| f(t,s,\psi,x) \| &\leq \| a_2 \|_\infty \left(\| \psi \|_{\mathcal{B}} + \| x \|\right),
\end{aligned}$$

for all $(t, s, \psi, x, y) \in I^2 \times \mathcal{B} \times X^2$. Moreover, a straightforward estimation using (i) shows that $g(\cdot)$ is a $D(A)$-valued, that $Ag(\cdot)$ is continuous and consequently that $g(\cdot)$ is completely continuous since the inclusion $i_c : [D(A)] \to X$ is compact. Also, $g(t, \cdot)$ is a bounded linear operator and $\max\{\| g(t, \cdot) \|_{\mathcal{L}(X,E)}, \| Ag(t, \cdot) \|_{\mathcal{L}(X)}\} \leq \Lambda_1^{\frac{1}{2}} r^{\frac{1}{2}}$, for every $t \in I$.

The next existence result is an immediate consequence of Theorem 3. We omit the proof.

Proposition 1 *Let $(\varphi, z) \in \mathcal{B} \times X$ be such that $\varphi(0) \in E$ and assume that the above conditions are satisfied. If*

$$\Lambda_1^{\frac{1}{2}} r^{\frac{1}{2}} (1 + a(1 + N_1)) + 2a^2 \max\{\|a_1\|_\infty, 1\}(1 + a\|a_2\|_\infty) < 1,$$

where $N_1 = \sup_{\theta \in I} \|AS(\theta)\|_{\mathcal{L}(E;X)}$, then there exists a unique mild solution of the neutral system (22)-(25).

To complete this section, we discuss briefly the existence of mild solutions for the neutral system

$$\frac{\partial}{\partial t}[\frac{\partial u(t,\xi)}{\partial t} + \int_{-r}^{0}\int_{0}^{\pi} b(\theta,\eta,\xi)u(t+\theta,\eta)d\eta d\theta] = \frac{\partial^2 u(t,\xi)}{\partial \xi^2}$$
$$+\int_0^t\int_0^\pi c(t,s,\eta,\xi)\left[u_s(-r,\eta) + u'(s,\eta) + \int_0^s a_2(s,\tau)(u_\tau(-r,\eta) + u'(\tau,\eta))d\tau\right] d\eta ds,$$
$$t \in I, \xi \in J, \qquad (26)$$

$$\begin{aligned} u(t,0) &= u(t,\pi) = 0, & t &\in I, & (27)\\ u_0 &= \varphi, & \varphi &\in \mathcal{B}, & (28)\\ \frac{\partial u(0,\xi)}{\partial t} &= z(\xi), & \xi &\in J, & (29)\end{aligned}$$

where $b(\cdot)$, $a_2(\cdot)$ satisfy the assumptions of the previous example, the functions $c(t, s, \eta, \xi)$, $\frac{\partial}{\partial \xi}c(t, s, \eta, \xi)$, $\frac{\partial^2}{\partial \xi^2} c(t, s, \eta, \xi)$ are continuous on $I^2 \times J^2$, $c(t, s, \eta, \pi) = c(t, s, \eta, 0) = 0$, for every $(t, s, \eta) \in I^2 \times J$. We consider the notation

$$\Lambda_2 := \sup\left\{\int_0^\pi\int_0^\pi \left(\frac{\partial^j}{\partial \xi^j}c(t,s,\eta,\xi)\right)^2 d\theta d\xi : j = 0,1,2,\, t,s \in I\right\}.$$

Let $g(\cdot), f(\cdot)$ be defined as before and $F : I^2 \times \mathcal{B} \times X^2 \to X$ be the function given by

$$F(t,s,\psi,x,y)(\xi) := \int_0^\pi c(t,s,\eta,\xi)\,[\psi(-r,\eta) + x(\eta) + y(\eta)]\, d\eta.$$

Using these identifications, we can describe system (26)-(29) as (1)-(3). It is easy to see that $AF(\cdot)$ is well defined, continuous and that $AF(t, s, \cdot)$ is a bounded linear operator with $\| AF(t, s, \cdot) \|_{\mathcal{L}(\mathcal{B}\times X\times X)} \leq \Lambda_2$, which in turn implies that $F(\cdot)$ is completely continuous since $i_c : [D(A)] \to X$ is compact. Moreover, the functions $f(\cdot), g(\cdot), F(\cdot)$ satisfy the assumptions of Theorem 2 with $W_F(s) = W_f(s) = W_g(s) = s$, $m_F(t,s) = \int_0^\pi \int_0^\pi c^2(t,s,\eta,\xi) d\eta d\xi$, $m_f = \| a_2 \|_\infty$ and $m_g = \Lambda_1^{\frac{1}{2}} r^{\frac{1}{2}}$. The next proposition is a consequence of Theorem 2.

Proposition 2 *Let $(\varphi, z) \in \mathcal{B} \times X$ be such that $\varphi(0) \in E$. Then, there exists a unique mild solution of the neutral system (26)-(29).*

References

[1] Balachandran, K.; Park, D. G.; Anthoni, S. Marshal Existence of solutions of abstract-nonlinear second-order neutral functional integrodifferential equations. *Comput. Math. Appl.* 46 (2003), no. 8-9, 1313–1324.

[2] Travis, C. C. and Webb, G. F., Compactness, regularity, and uniform continuity properties of strongly continuous cosine families, *Houston J. Math.* 3 (4) (1977) 555-567.

[3] Hale, J. K. and Verduyn Lunel, S. M., Introduction to Functional-Differential Equations. Applied Mathematical Sciences, 99. Springer-Verlag, New York, 1993.

[4] Lakshmikantham, V., Wen, L. and Zhang, B., Theory of Differential Equations with Unbounded Delay. Kluwer Acad. Publ., Dordrecht, 1994.

[5] Kolmanovskii, V. and Myshkis, A., Introduction to the Theory and Applications of Functional Differential Equations. Kluwer Acad. Publ., Dordrecht, 1999.

[6] Adimy, M., Ezzinbi, K.and Laklach, M., Existence of solutions for a class of partial neutral differential equations. *C. R. Acad. Sci. Paris Sér. I Math.* 330 (2000), no. 11, 957–962.

[7] Adimy M., Bouzahir, H. and Ezzinbi K., Existence and stability for some partial neutral functional differential equations with infinite delay. *J. Math. Anal. Appl.* In Press, Corrected Proof, Available online 6 May 2004,

[8] Adimy, M., Ezzinbi, K., A class of linear partial neutral functional-differential equations with nondense domain. *J. Diff. Eqns.* 147 (1998), no. 2, 285–332.

[9] Adimy, M.; Ezzinbi, K. Strict solutions of nonlinear hyperbolic neutral differential equations. *Appl. Math. Lett.* 12 (1999), no. 1, 107-112.

[10] Datko, R., Linear autonomous neutral differential equations in a Banach space. *J. Diff. Eqns.* 25 (1977), no. 2, 258–274.

[11] Hale, J. K., Partial neutral functional-differential equations. *Rev. Roumaine Math Pures Appl.* 39 (1994), no. 4, 339-344.

[12] Wu, J. and Xia, H., Rotating waves in neutral partial functional-differential equations. *J. Dynam. Diff. Eqns.* 11 (1999), no. 2, 209–238.

[13] Wu, J. and Xia, H., Self-sustained oscillations in a ring array of coupled lossless transmission lines. *J. Diff. Eqns.* 124 (1996), no. 1, 247–278.

[14] Hernández, E. and Henríquez, H. R., Existence of periodic solutions of partial neutral functional-differential equations with unbounded delay. *J. Math. Anal. Appl.* 221 (1998), no. 2, 499–522.

[15] Hernández, E. and Henríquez, H. R., Existence results for partial neutral functional-differential equations with unbounded delay. *J. Math. Anal. Appl.* 221 (1998), no. 2, 452–475.

[16] Hernández, Eduardo, Existence Results for Partial Neutral Integrodifferential Equations with Unbounded Delay. *Journal of Mathematical Analysis and Applications,* 292, (2004), no. 1, 194-210.

[17] Balachandran, K.; Anthoni, S. M., Existence of solutions of second order neutral functional-differential equations. *Tamkang J. Math.* 30 (1999), no. 4, 299–309.

[18] Balachandran, K.; Anthoni, S. Marshal Controllability of second-order semilinear neutral functional differential systems in Banach spaces. *Comput. Math. Appl.* 41 (2001), no. 10-11, 1223–1235.

[19] Fattorini, H. O., Second Order Linear Differential Equations in Banach Spaces, North-Holland Mathematics Studies, Vol. 108, North-Holland, Amsterdam, 1985.

[20] Travis, C. C. and Webb, G. F., Cosine families and abstract nonlinear second order differential equations. *Acta Math.Acad. Sci. Hungaricae,* 32 (1978) 76-96.

[21] Kisyński, J., On cosine operator functions and one parameter group of operators, *Studia Math.* 49 (1972), 93-105.

[22] Hino, Y., Murakami, S. and Naito, T., Functional-Differential Equations with Infinite Delay. Lecture Notes in Mathematics, 1473. Springer-Verlag, Berlin, 1991.

[23] Granas, A. and Dugundji, J., Fixed Point Theory. Springer-Verlag, New York, 2003.

[24] Martin, R. H., Nonlinear Operators and Differential Equations in Banach Spaces, Robert E. Krieger Publ. Co., Florida, 1987.

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Resumo

Neste paper mostramos a existência de soluções fracas para uma classe de equações integrodiferencias do tipo neutro descritas na forma

$$\begin{aligned}
\frac{d}{dt}\left[x'(t) - g(t, x_t)\right] &= Ax(t) + \int_0^t F(t, s, x_s, x'(s), \int_0^s f(s, \tau, x_\tau, x'(\tau))d\tau)ds, && (1)\\
x(0) &= \varphi \in \mathcal{B}, && (2)\\
x'(0) &= z \in X, && (3)
\end{aligned}$$

sendo A o gerador infinitesimal de uma função cosseno de operadores lineares sobre um espaço de Banach X, $\mathcal{B}$ um espaço de fase definido de maneira axiomática e $f, g, F : [0, a] \times \mathcal{B} \to X$ funções apropriadas.

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