



UNIVERSIDADE DE SÃO PAULO
Instituto de Ciências Matemáticas e de Computação
ISSN 0103-2577

DEDALUS - Acervo - ICMC



**FAST LOOPS ON SEMI-WEIGHTED HOMOGENEOUS
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Nº 325

NOTAS DO ICMC
SÉRIE MATEMÁTICA



São Carlos – SP
Mar./2010

Class.	<u>Notas SMA - Nº 325</u>
Cutter	<u>F363 fl</u>
Tombo	<u>39.384</u>
Syno	

FAST LOOPS ON SEMI-WEIGHTED HOMOGENEOUS HYPERSURFACE SINGULARITIES

ALEXANDRE FERNANDES

ABSTRACT. We show the existence of $(1 + \frac{w_2}{w_3})$ -fast loops on semi-weighted homogeneous hypersurface singularities with weights $w_1 \geq w_2 > w_3$. In particular we show that semi-weighted homogeneous hypersurface singularities have metrical conical structure only if its two low weights are equal.

1. INTRODUCTION

Let $X \subset \mathbb{R}^n$ be a subanalytic set with a singularity at x . It is well-known for small real numbers $\epsilon > 0$ that there exists a homeomorphism from the Euclidean ball $B(x, \epsilon)$ to itself which maps $X \cap B(x, \epsilon)$ onto the straight cone over $X \cap S(x, \epsilon)$ with vertex at x . The homeomorphism h is called a *topological conical structure* of X at x and, since John Milnor proved the existence of topological conical structure for algebraic complex hypersurfaces with an isolated singularity [11], some authors say ϵ is a *Milnor radius* of X at x . Some developments of a Lipschitz geometry of complex algebraic singularities come from the following question: given an algebraic subset $X \subset \mathbb{C}^n$ with an isolated singularity at x , is there $\epsilon > 0$ such that $X \cap B(x, \epsilon)$ is bi-Lipschitz homeomorphic to the cone over $X \cap S(x, \epsilon)$ with vertex at x ? When we have a positive answer for this question we say that (X, x) *admits a metrical conical structure*. Some motivations for this question were given in [4], [7] and, in the same papers, the above question was answered negatively. The strategy used in [7] to show that some examples of complex algebraic surface singularities do not admit a metrical conical structure was estimating a homotopic version of the first characteristic exponent for weighted homogeneous surface singularities (see [2])

Date: March 14, 2010.

1991 Mathematics Subject Classification. 14B05, 14J17, 32S25.

Key words and phrases. bi-Lipschitz, Complex Singularity.

This paper was written during my posdoctoral stage at Universidade de São Paulo and this research was partially supported by CNPq grant N 150578/2009-1.

or [3] for a definition of characteristic exponent). In this paper we compute this exponent for some semi-weighted homogeneous surface singularities.

2. PRELIMINARIES

2.1. Inner metric. Given an arc $\gamma: [0, 1] \rightarrow \mathbb{R}^n$, we remember that the *length* of γ is defined by

$$l(\gamma) = \inf \left\{ \sum_{i=1}^m |\gamma(t_i) - \gamma(t_{i-1})| : 0 = t_0 < t_1 < \dots < t_{m-1} < t_m = 1 \right\}.$$

Let $X \subset \mathbb{R}^n$ be a subanalytic connected subset. It is well-known that the function

$$d_X: X \times X \rightarrow [0, +\infty)$$

defined by

$$d_X(x, y) = \inf \{ l(\gamma) : \gamma: [0, 1] \rightarrow X; \gamma(0) = x, \gamma(1) = y \}$$

is a metric on X , so-called *inner metric* on X .

Theorem 2.1 (Pancake Decomposition [10]). *Let $X \subset \mathbb{R}^n$ be a subanalytic connected subset. Then, there exist $\lambda > 0$ and $X_1, \dots, X_m$ subanalytic subsets such that:*

- a. $X = \bigcup_{i=1}^m X_i$,
- b. $d_X(x, y) \leq \lambda|x - y|$ for any $x, y \in X_i$, $i = 1, \dots, m$.

2.2. Horn exponents. Let $\beta \geq 1$ be a rational number. The germ of

$$H_\beta = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^\beta, z \geq 0\}$$

at origin $\in \mathbb{R}^3$ is called a β -horn.

By results of [1], we conclude that a β_1 -horn is bi-Lipschitz equivalent, with respect to the inner metric, to a β_2 -horn if, and only if $\beta_1 = \beta_2$. Let $\Omega \subset \mathbb{R}^n$ be a 2-dimensional subanalytic set. Let $x_0 \in \Omega$ be a point such that Ω is a topological 2-dimensional manifold without boundary near x_0 .

Theorem 2.2. [1] *There exists a unique rational number $\beta \geq 1$ such that the germ of Ω at x_0 is bi-Lipschitz equivalent, with respect to the inner metric, to a β -horn.*



The number β is called *the horn exponent of Ω at x_0* . We use the notation $\beta(\Omega, x_0)$. By Theorem 2.2, $\beta(\Omega, x_0)$ is a complete intrinsic bi-Lipschitz invariant of germ of subanalytic sets which are topological 2-dimensional manifold without boundary. In the following, we show a way to compute horn exponents.

According to [2], $\beta(\Omega, x_0) + 1$ is the volume growth number of Ω at x_0 , i. e.

$$\beta(\Omega, x_0) + 1 = \lim_{r \rightarrow 0+} \frac{\log \mathcal{H}^2[\Omega \cap B(x_0, r)]}{\log r}$$

where $\mathcal{H}^2$ denotes the 2-dimensional Hausdorff measure with respect to Euclidean metric on $\mathbb{R}^n$.

2.3. Order of contact of arcs. Let $\gamma_1: [0, \epsilon) \rightarrow \Omega$ and $\gamma_2: [0, \epsilon) \rightarrow \Omega$ be two continuous semianalytic arcs with $\gamma_1(0) = \gamma_2(0) = x_0$ and not identically equal to x_0 . We suppose that the arcs are parameterized in the following way:

$$\|\gamma_i(t) - x_0\| = t, \quad i = 1, 2.$$

Let $\rho(t)$ be a function defined as follows: $\rho(t) = \|\gamma_1(t) - \gamma_2(t)\|$. Since ρ is a subanalytic function there exist numbers $\lambda \in \mathbb{Q}$ and $a \in \mathbb{R}$, $a \neq 0$, such that

$$\rho(t) = at^\lambda + o(t^\lambda).$$

The number λ is called *an order of contact of γ_1 and γ_2* . We use the notation $\lambda(\gamma_1, \gamma_2)$ (see [5]).

Let K be the field of germs of subanalytic functions $f: (0, \epsilon) \rightarrow \mathbb{R}$. Let $\nu: K \rightarrow \mathbb{R}$ be a canonical valuation on K . Namely, if $f(t) = \alpha t^\beta + o(t^\beta)$ with $\alpha \neq 0$ we put $\nu(f) = \beta$.

Lemma 2.3. *Let γ_1, γ_2 be a pair of semianalytic arcs such that $\gamma_1(0) = \gamma_2(0) = x_0$ and $\gamma_i \neq x_0$ ($i = 1, 2$). Let $\tilde{\gamma}_1(\tau)$ and $\tilde{\gamma}_2(\tau)$ be semianalytic parameterizations of γ_1 and γ_2 such that $\|\tilde{\gamma}_i(\tau) - x_0\| = \tau + o_i(\tau)$, $i = 1, 2$. Let $l(\tau) = \|\tilde{\gamma}_1(\tau) - \tilde{\gamma}_2(\tau)\|$. Then $\nu(l(\tau)) \leq \lambda(\gamma_1, \gamma_2)$.*

The following result is an alternative way to compute horn exponents of germ of subanalytic sets which are topological 2-dimensional manifold without boundary.

Theorem 2.4. *Let $\Omega \subset \mathbb{R}^n$ be a 2-dimensional subanalytic set. Let $x_0 \in \Omega$ be a point such that Ω is a topological 2-dimensional manifold without boundary near x_0 . Then $\beta(\Omega, x_0) = \inf\{\lambda(\gamma_1, \gamma_2) : \gamma_1, \gamma_2 \text{ are semianalytic arcs on } \Omega \text{ with } \gamma_1(0) = \gamma_2(0) = x_0\}$.*

Lemma 2.3 and Theorem 2.4 were proved in [6].

3. FAST LOOPS

Let $X \subset \mathbb{R}^n$ be a subanalytic set with a singularity at x . Let $\epsilon > 0$ be a Milnor radius of X at x and let us denote by X^* the set $X \cap B(x, \epsilon) \setminus \{x\}$. Given a positive real number α , a continuous map $\gamma: S^1 \rightarrow X^*$ is called a α -fast loop if there exists a homotopy $H: S^1 \times [0, 1] \rightarrow X \cap B(x, \epsilon)$ such that

- (1) $H(\theta, 0) = x$ and $H(\theta, 1) = \gamma(\theta)$, $\forall \theta \in S^1$,
- (2) $\lim_{r \rightarrow 0^+} \frac{1}{r^\alpha} \mathcal{H}^2(Im(H) \cap B(x, r)) = 0$ for each $0 < \alpha < \alpha$,

where $Im(H)$ denotes the image of H .

Given a subanalytic set X and a singular point $x \in X$, according to [2], there exists a positive number c such that any α -fast loop $\gamma: S^1 \rightarrow X^*$ with $\alpha > c$ is necessarily homotopically trivial. Such a number c is called *distinguished for (X, x)* . We define the v invariant in the following way:

$$v(X, x) = \inf\{c : c \text{ is distinguished for } (X, x)\}.$$

The number $v(X, x)$ defined above is a homotopic version of the first characteristic exponent for the local metric homology presented in [2].

Example 3.1. Let $K \subset \mathbb{R}^n$ be a straight cone over a Nash submanifold $N \subset \mathbb{R}^n$, with vertex at p . Then every loop $\gamma: S^1 \rightarrow K^*$ is a 2-fast loop. Moreover, if $\alpha > 2$, then each α -fast loop $\gamma: S^1 \rightarrow K^*$ is homotopically trivial. We can sum up it saying $v(K, p) = 2$.

Proposition 3.2. *Let (X, x) and (Y, y) be subanalytic germs. If there exists a germ of a bi-Lipschitz homeomorphism, with respect to inner metric, between (X, x) and (Y, y) , then $v(X, x) = v(Y, y)$.*



Proof. Let $f: (X, x) \rightarrow (Y, y)$ be a bi-Lipschitz homeomorphism, with respect to the inner metric. Given $A \subset X$, let us denote $\tilde{A} = f(A)$. In this case, $A = f^{-1}(\tilde{A})$, where f^{-1} denotes the inverse map of $f: (X, x) \rightarrow (Y, y)$.

Claim. There are positive constants $k_1, k_2, \lambda_1, \lambda_2$ such that

$$\frac{1}{k_1} \mathcal{H}^2(\tilde{A} \cap B(y, \frac{r}{\lambda_2})) \leq \mathcal{H}^2(A \cap B(x, r)) \leq k_2 \mathcal{H}^2(\tilde{A} \cap B(y, \lambda_1 r)).$$

In fact, using Pancake Decomposition Theorem (see Subsection 2.1) and using that f and f^{-1} are Lipschitz maps, we obtain positive constants λ_1, λ_2 such that

$$f(A \cap B(x, r)) \subset (\tilde{A} \cap B(y, \lambda_1 r)) \text{ and } f(\tilde{A} \cap B(y, r)) \subset (A \cap B(x, \lambda_2 r))$$

and we also obtain positive constants k_1, k_2 such that

$$\mathcal{H}^2(f(A \cap B(x, r))) \leq k_1 \mathcal{H}^2(\tilde{A} \cap B(y, \lambda_1 r)) \text{ and } \mathcal{H}^2(f^{-1}(\tilde{A} \cap B(y, r))) \leq k_2 \mathcal{H}^2(A \cap B(x, \lambda_2 r)).$$

Our claim follows from this two inequalities and the two inclusions above.

Now, we use this claim to show that given $\alpha > 0$, a loop $\gamma: S^1 \rightarrow X \setminus \{x\}$ is an α -fast loop if, and only if, $f \circ \gamma: S^1 \rightarrow Y \setminus \{y\}$ is an α -fast loop. In fact, let $\gamma: S^1 \rightarrow X \setminus \{x\}$ be a loop and $H: S^1 \times [0, 1] \rightarrow X$ a homotopy such that $H(\theta, 0) = x$ and $H(\theta, 1) = \gamma(\theta)$, $\forall \theta \in S^1$. Thus, $f \circ \gamma: S^1 \rightarrow Y \setminus \{y\}$ is a loop and $f \circ H: S^1 \times [0, 1] \rightarrow X$ is a homotopy such that $f \circ H(\theta, 0) = x$ and $f \circ H(\theta, 1) = f \circ \gamma(\theta)$, $\forall \theta \in S^1$. Let us denote $A = \text{Im}(H)$ and $\tilde{A} = \text{Im}(f \circ H)$, i. e., $\tilde{A} = f(A)$. Given $0 < a < \alpha$, by the above claim, we have that

$$\lim_{r \rightarrow 0^+} \frac{1}{r^a} \mathcal{H}^2(A \cap B(x, r)) = 0$$

if, and only if,

$$\lim_{r \rightarrow 0^+} \frac{1}{r^a} \mathcal{H}^2(\tilde{A} \cap B(y, r)) = 0.$$

In another words, it was shown that $\gamma: S^1 \rightarrow X \setminus \{x\}$ is an α -fast loop if, and only if, $f \circ \gamma: S^1 \rightarrow Y \setminus \{y\}$ is an α -fast loop, hence $v(X, x) = v(Y, y)$. $\square$

Corollary 3.3. *Let $X \subset \mathbb{R}^n$ be a subanalytic set and $x \in X$ an isolated singular point. If $v(X, x) > 2$, then X does not admit a metrical conical structure at x .*

Proof. Let N be the intersection $X \cap S(x, \epsilon)$ where $\epsilon > 0$ is chosen sufficiently small. Since x is an isolated singular point of X , we have $N \subset \mathbb{R}^n$ is a Nash

submanifold. If X has metrical conical structure at x , $X \cap B(x, \epsilon)$ must be bi-Lipschitz homeomorphic (with respect to the inner metric) to the straight cone over N with vertex at x . Thus, it follows from Proposition 3.2 that $v(X, x) = 2$. $\square$

4. SEMI-WEIGHTED HOMOGENEOUS HYPERSURFACE SINGULARITIES

Remind that a polynomial function $f: \mathbb{C}^3 \rightarrow \mathbb{C}$ is called *semi-weighted homogeneous* of degree $d \in \mathbb{N}$ with respect to the weights $w_1, w_2, w_3 \in \mathbb{N}$ if f can be present in the following form: $f = h + \theta$ where h is a weighted homogeneous polynomial of degree d with respect to the weights w_1, w_2, w_3 , the origin is an isolated singularity of h and θ contains only monomials $x^m y^n z^l$ such that $w_1 m + w_2 n + w_3 l > d$.

An algebraic surface $S \subset \mathbb{C}^3$ is called *semi-weighted homogeneous* if there exists a semi-weighted homogeneous polynomial $f = h + \theta$ such that $S = \{(x, y, z) \in \mathbb{C}^3 : f(x, y, z) = 0\}$. The set $S_0 = \{(x, y, z) \in \mathbb{C}^3 : h(x, y, z) = 0\}$ is called a *weighted approximation* of S .

Theorem 4.1. *Let $S \subset \mathbb{C}^3$ be a semi-weighted homogeneous algebraic surface with an isolated singularity at origin $0 \in \mathbb{C}^3$. If the weights of S satisfy $w_1 \geq w_2 > w_3$, then $v(S, 0) \geq 1 + \frac{w_2}{w_3}$.*

Proof. Let us consider a family of functions defined as follows:

$$F(X, u) = h(X) + u\theta(X),$$

where $u \in [0, 1]$, $X = (x, y, z)$, and let us denote $F_u(X) = F(X, u)$. Let $V(X, u)$ be the vector field defined by:

$$V(X, u) = \frac{\theta(X)}{N^*F_u(X)} W(X, u)$$

where

$$N^*F_u(X) = \left(\frac{\partial F}{\partial x}(X, u)\right)^{2\alpha_a} + \left(\frac{\partial F}{\partial y}(X, u)\right)^{2\alpha_b} + \left(\frac{\partial F}{\partial z}(X, u)\right)^{2\alpha_c},$$

$\alpha_a = (d - b)(d - c)$, $\alpha_b = (d - a)(d - c)$, $\alpha_c = (d - a)(d - b)$ and

$$W(X, u) = \left(\frac{\partial F}{\partial x}(X, u)\right)^{2\alpha_a-1} \frac{\partial}{\partial x} + \left(\frac{\partial F}{\partial y}(X, u)\right)^{2\alpha_b-1} \frac{\partial}{\partial y} + \left(\frac{\partial F}{\partial z}(X, u)\right)^{2\alpha_c-1} \frac{\partial}{\partial z}.$$

It was shown, by L. Fukui and L. Paunescu [9], that the flow of this vector field gives a modified analytic trivialization [9] of the family $F^{-1}(0)$. In particular, the

map defined by

$$\Phi(X) = X + \int_0^1 V(X, u) du$$

is a homeomorphism between $(S, 0)$ and $(\tilde{S}, 0)$ which defines a correspondence between subanalytic arcs in $(S, 0)$ and in $(\tilde{S}, 0)$.

Proposition 4.2. *Let $\gamma: [0, \epsilon) \rightarrow V$; $\gamma(0) = 0$, be a subanalytic continuous arc parameterized by $\gamma(t) = (t^{w_1}x(t), t^{w_2}y(t), t^{w_3}z(t))$. Then $\Phi(\gamma(t)) = \gamma(t) + \eta(t)$ such that $\eta(t) = (\eta_1(t), \eta_2(t), \eta_3(t))$ and $\nu(|\eta_i|) > w_i$ for $i = 1, 2, 3$.*

Proof of the proposition. It is easy to see that there exists a constant $\lambda > 0$ such that $|\frac{\partial F_u}{\partial x}(\gamma(t))| \leq \lambda t^{d-a}$, $|\frac{\partial F_u}{\partial y}(\gamma(t))| \leq \lambda t^{d-b}$ and $|\frac{\partial F_u}{\partial z}(\gamma(t))| \leq \lambda t^{d-c}$.

It was shown, by M. Ruas and M. Saia ([12], Lemma 3, p.93), that there exists a constant λ_1 such that

$$N^*F_u(\gamma(t)) \geq \lambda_1 N^*h(\gamma(t)).$$

Since $N^*h(\gamma(t)) = t^{2k}N^*h(x(t), y(t), z(t))$, there exists a constant λ_2 such that

$$N^*F_u(\gamma(t)) \geq \lambda_2 t^{2k}.$$

By hypothesis, $\lim_{t \rightarrow 0+} \theta(\gamma(t))t^{-d} = 0$. Thus,

$$\begin{aligned} |\eta_1(t)|t^{-a} &\leq t^{-a} \int_0^1 \frac{|\theta(\gamma(t))|}{N^*F_u(\gamma(t))} \left| \frac{\partial F_u}{\partial x}(\gamma(t)) \right|^{2\alpha_a-1} du \\ &\leq \frac{\lambda^{2\alpha_a-1}}{\lambda_2} |\theta(\gamma(t))|t^{-d} \end{aligned}$$

$$\therefore \lim_{t \rightarrow 0+} \eta_1(t)t^{-a} = 0;$$

$$\begin{aligned} |\eta_2(t)|t^{-b} &\leq t^{-b} \int_0^1 \frac{|\theta(\gamma(t))|}{N^*F_u(\gamma(t))} \left| \frac{\partial F_u}{\partial y}(\gamma(t)) \right|^{2\alpha_b-1} du \\ &\leq \frac{\lambda^{2\alpha_b-1}}{\lambda_2} |\theta(\gamma(t))|t^{-d} \end{aligned}$$

$$\therefore \lim_{t \rightarrow 0+} \eta_2(t)t^{-b} = 0;$$

$$\begin{aligned} |\eta_3(t)|t^{-c} &\leq t^{-c} \int_0^1 \frac{|\theta(\gamma(t))|}{N^*F_u(\gamma(t))} \left| \frac{\partial F_u}{\partial z}(\gamma(t)) \right|^{2\alpha_c-1} du \\ &\leq \frac{\lambda^{2\alpha_c-1}}{\lambda_2} |\theta(\gamma(t))|t^{-d} \end{aligned}$$

$$\therefore \lim_{t \rightarrow 0^+} \eta_3(t) t^{-c} = 0. \quad \square$$

According to Lemma 1 of [7], we can take an essential loop Γ from S^1 to the link of the weighted homogeneous approximation of $(X, 0)$ of the form:

$$\Gamma(\theta) = (x(\theta), y(\theta), 1).$$

Then, $H: [0, 1] \times S^1 \rightarrow X$ defined by

$$H(r, \theta) = \Phi(r^{\frac{w_1}{w_3}} x(\theta), r^{\frac{w_2}{w_3}} y(\theta), r)$$

is a subanalytic homotopy satisfying: $H(0, \theta) = x$ and $H(1, \theta) = \Gamma(\theta)$. We are going to show the image of H ($Im(H) = \Omega$) has volume growth number at origin bigger than or equal to $1 + \frac{w_2}{w_3}$. Actually, since the volume growth number of Ω at 0 is $1 + \beta(\Omega, 0)$, we are going to show that $\beta(\Omega, 0)$ is bigger than or equal to $\frac{w_2}{w_3}$. So, let us consider two arcs γ_1 and γ_2 on $(\Omega, 0)$. We can parameterize these arcs in the following way:

$$\gamma_i(t) = H(t, \theta_i(t)), \quad i = 1, 2.$$

By Lemma 2.3, we have

$$\lambda(\gamma_1, \gamma_2) \geq \nu(|\gamma_1(t) - \gamma_2(t)|)$$

and by Proposition 4.2, we have

$$\nu(|\gamma_1(t) - \gamma_2(t)|) \geq \frac{w_2}{w_3}.$$

Therefore, we can use Theorem 2.4 to get $\beta(\Omega, 0) \geq \frac{w_2}{w_3}$. $\square$

Corollary 4.3. *Let $S \subset \mathbb{C}^3$ be a semi-weighted homogeneous algebraic surface with an isolated singularity at origin $0 \in \mathbb{C}^3$. If the two low weights of S are unequal, then the germ $(S, 0)$ does not admit a metrical conical structure.*

Proof. Let $w_1 \geq w_2 \geq w_3 > 0$ be the weights of S . By hypothesis, $w_2 > w_3$. It follows from Theorem 4.1,

$$\nu(S, 0) \geq 1 + \frac{w_2}{w_3} > 2.$$

Finally, by Corollary 3.3, $(S, 0)$ does not admit a metrical conical structure. $\square$

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